Mock TMUA Set A: Paper 1

20 questions

75 minutes

No calculator allowed

1. Find the value of $\displaystyle\int_1^4 \frac{2-x^2}{x\sqrt{x}}\,dx.$

A $-\frac{16}{3}$ B $-\frac{10}{3}$ C $-\frac{8}{3}$ D 0 E $\frac{2}{3}$

$$\int_1^4 2x^{-3/2} - x^{1/2}\,dx = \left[-4x^{-1/2} - \tfrac{2}{3}x^{3/2}\right]_1^4$$

$$= \left(-4\left(\tfrac{1}{2}\right) - \tfrac{2}{3}(8)\right) - \left(-4 - \tfrac{2}{3}\right) = -2 - \tfrac{16}{3} + 4 + \tfrac{2}{3} = 2 - \tfrac{14}{3} = -\tfrac{8}{3}$$

2. A cuboid has sides of length $x, \sqrt{2}x$ and $2x$ measured in cm.

The volume of the cuboid is numerically equal to twice the total surface area. What is the value of x ?

A 10 B $6+2\sqrt{2}$ C 5 D $\frac{5}{2}$ E $\frac{3+\sqrt{2}}{2}$

$$V = 2\sqrt{2}x^3 = 2\times 2\left(\sqrt{2} + 2 + 2\sqrt{2}\right)x^2$$

$$2\sqrt{2}x = 4\sqrt{2} + 8 + 8\sqrt{2} = 12\sqrt{2} + 8$$

$$x = \frac{6\sqrt{2}+4}{\sqrt{2}} = 6 + 2\sqrt{2}$$

3. The function f is given by $f(x) = \dfrac{(2x-1)(3x-2)}{2\sqrt{x^3}}$ find the value of $f'(1)$

A $\frac{1}{2}$ B $-\frac{1}{2}$ C $\frac{7}{4}$ D $\frac{19}{4}$ E $\frac{13}{2}$

$$f(x) = \frac{6x^2 - 7x + 2}{2x^{3/2}} = 3x^{1/2} - \tfrac{7}{2}x^{-1/2} + x^{-3/2}$$

$$f'(x) = \tfrac{3}{2}x^{-1/2} + \tfrac{7}{4}x^{-3/2} - \tfrac{3}{2}x^{-5/2}$$

$$f'(1) = \tfrac{3}{2} + \tfrac{7}{4} - \tfrac{3}{2} = \tfrac{7}{4}$$

4. What is the sum of all real solutions of the equation $2\sqrt{a} + \frac{7}{\sqrt{a}} = 9 - \frac{6}{a}$

A $4\frac{1}{2}$ B 5 C $12\frac{1}{2}$ D 13 E $13\frac{1}{4}$

$2a^{3/2} + 7a^{1/2} = 9a - 6$

$2a^{3/2} - 9a + 7a^{1/2} + 6 = 0$

$\sqrt{a} = x > 0$

$2x^3 - 9x^2 + 7x + 6 = 0$

$(x-2)(2x^2 - 5x - 3) = 0$

$(x-2)(x-3)(2x+1) = 0$

$x = 2, 3, \cancel{-1/2}$

$a = x^2 = 4, 9$

5. The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2p)$ is parallel to the line with equation $2x + 3y + 1 = 0$

What is the value of p ?

A -1 B $\frac{2}{3}$ C $\frac{5}{4}$ D 2 E 5

gradient $= \frac{p-1-2p}{p-1+p} = \frac{-p-1}{2p-1} = -\frac{2}{3}$

$3y = -2x - 1$

$y = -\frac{2}{3}x - \frac{1}{3}$

$3p + 3 = 4p - 2$

$5 = p$

6. Anna and Ben are 360 miles apart. Anna travels towards Ben at a constant speed of 25 mph, and Ben travels towards Anna at a constant speed of 65mph. How many miles apart are they, 90 minutes before they meet?

A 40

B 85

C 135

D 225

x $360 - x$

A B

$\frac{x}{25} = \frac{360-x}{65}$

$13x = 1800 - 5x$

$18x = 1800$

$x = 100$

D

S T

OR

$T = 1.5$

$S = 90$

$D = 90 \times 1.5$

$= 135$

A: $T = 1.5$ $S = 25$

$D = 37.5$

B

$D = 1.5 \times 65 = 97.5$

$37.5 + 97.5 = 135$

7. A sequence of terms is given by $a_n = \dfrac{n+1}{n+3}$

What is the product of the first 40 terms?

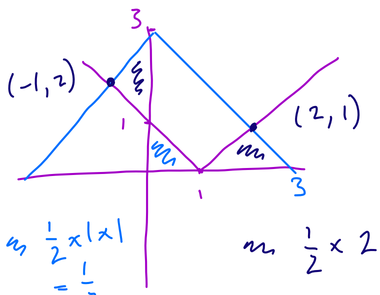
A $\frac{3}{946}$ B $\frac{1}{301}$ C $\frac{1}{287}$ D $\frac{1}{7}$ E $\frac{1}{2}$

$a_1 = \frac{2}{4}$ $a_2 = \frac{3}{5}$ $a_3 = \frac{4}{6}$

$\frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \ldots \times \frac{40}{42} \times \frac{41}{43} = \frac{2 \times 3}{42 \times 43} = \frac{1}{7 \times 43}$

8. Find the area between the curves given by $y = |x-1|$ and $y = 3 - |x|$?

A 1 B $\frac{3}{2}$ C 3 D 4 E 6



$\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

$\frac{1}{2} \times 2 \times 1 = 1$

$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

$\frac{9}{2} - \frac{1}{2} = 4$

or

$\sqrt{1^2 + 1^2} = \sqrt{2}$

$\sqrt{2^2 + 2^2} = \sqrt{8}$

Area of rectangle $= \sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$

9. What is the maximum value of the following expression

$$-3x^2 + 12x - 2y^2 - 12y - 39?$$

A -39 B -9 C 0 D 9 E 39

Max of $-(3x^2 - 12x + 2y^2 + 12y + 39) = -$ Min of $(\quad)$

$3(x^2 - 4x) + 2(y^2 + 6y) + 39$

$3[(x-2)^2 - 4] + 2[(y+3)^2 - 9] + 39$

$3(x-2)^2 - 12 + 2(y+3)^2 - 18 + 39$

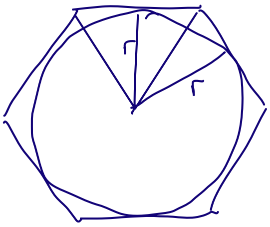
$3(x-2)^2 + 2(y+3)^2 + 9$ Min $= 9$

Max $= -9$

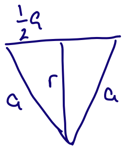
10. A circle is drawn inside a regular hexagon so that the circle touches each side of the hexagon.

What fraction of the hexagon is covered by the circle?

A $\frac{\sqrt{3}\pi}{2}$ B $\frac{\sqrt{3}\pi}{6}$ C $\frac{3\sqrt{3}}{2\pi}$ D $\frac{\sqrt{3}}{\pi}$ E $\frac{2\sqrt{3}\pi}{9}$



Circle: πr^2



$\frac{a^2}{4} + r^2 = a^2$

$r^2 = \frac{3a^2}{4}$ $a^2 = \frac{4r^2}{3}$

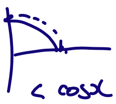
$\frac{\pi}{2\sqrt{3}} = \frac{\sqrt{3}\pi}{6}$

Hexagon: $6 \times \frac{1}{2} a^2 \frac{\sqrt{3}}{2} = \cancel{6}^{2} \times \frac{\cancel{4}^{2}}{\cancel{3}} r^2 \times \frac{\sqrt{3}}{\cancel{8}} = 2\sqrt{3} r^2$

11. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

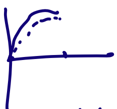
A $\int_0^{\frac{\pi}{2}} cosx \ \ dx$ B $\int_0^{\frac{\pi}{2}} cos^2 x \ \ dx$

C $\int_0^{\frac{\pi}{2}} \sqrt{sinx} \ \ dx$ D $\int_0^{\frac{\pi}{2}} sin^4 x \ \ dx$ E $\int_0^{\frac{\pi}{2}} 1 - \frac{2}{\pi}x \ \ dx$



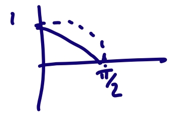
$< \cos x$

B, D $<$ A



$> \cos x$

C $>$ A



E $< \cos x =$ A

12. $f(x)$ is a quadratic function. Given that the graph of $y = f(x)$ passes through the point $(1, -5)$ and has a turning point at $(-1,3)$, which of the following is an expression for $f(x)$

A $-x^2 - 4x + 1$
B $-2x^2 - 4x + 1$
C $-x^2 - 4x - 3$
D $2x^2 - 4x - 3$
E $-2x^2 + 4x + 1$

$y = k(x+1)^2 + 3$

$(1,-5)$ $4k + 3 = -5$

$k = -2$

$= -2(x+1)^2 + 3$

$-2x^2 - 4x + 1$

13. Find the coefficient of x^3y^2 in the expansion of $(1 + x + y^2)^5$

A 5

B 10

C 20

D 30

E 60

$\underline{5}(1+x)^4(y^2)^1$

$(1+x)^4 = 1 + 4x + 6x^2 + \underline{4x^3} + x^4$

$5 \times 4 = 20$

OR Trinomial

$(1)^1 (x)^3 (y^2)^1 \times \frac{5!}{1!\,3!\,1!} = 5 \times 4 = 20$

14. Find the complete set of values of x with $\pi \leq x \leq 2\pi$ such that

$$(\sqrt{3} - 2cosx)\, sinx \geq 0$$

A $\pi \leq x \leq \frac{7\pi}{6}$

B $\pi \leq x \leq \frac{5\pi}{3}$

C $\pi \leq x \leq \frac{11\pi}{6}$

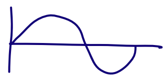
D $\frac{7\pi}{6} \leq x \leq 2\pi$

E $\frac{5\pi}{3} \leq x \leq 2\pi$

F $\frac{11\pi}{6} \leq x \leq 2\pi$

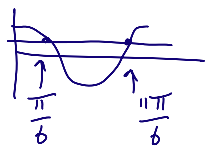
$\pi \leq x \leq 2\pi$

$\sin x \leq 0$



so need $\sqrt{3} - 2\cos x \leq 0$

$\cos x \geq \frac{\sqrt{3}}{2}$



15. The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers *n*:

$$F(n) = \frac{1}{n}\int_0^n (n - \frac{3}{2}x)\ dx \qquad G(n) = \sum_{r=0}^{n} F(r)$$

What is the largest positive integer *n* such that $G(n) < 69$?

$\frac{n(n+1)}{8} < 69$

$n(n+1) < 552$

$n = 23$ too big

24×23: $72 + 480 = 552$

A 20 B 21 C 22 D 23 E 24

$F(n) = \frac{1}{n}\int_0^n n - \frac{3}{2}x\ dx = \frac{1}{n}\left[nx - \frac{3}{4}x^2\right]_0^n = \frac{1}{n}\left(n^2 - \frac{3}{4}n^2\right) = \frac{1}{4}n$

$G(n) = \sum_{r=1}^{n} \frac{1}{4}n = \frac{1}{4}(1 + 2 + \dots + n) = \frac{1}{4} \times \frac{1}{2}n(n+1) = \frac{n(n+1)}{8}$

16. The graph of $y = log_{10}x$ is translated in the negative *y*-direction by 2 units.

This translation is equivalent to a stretch of factor *k* parallel to the *x*-axis?

What is the value of *k*?

A 0.01 B $log_{10}2$ C 0.5 D $log_2 10$ E 100

$\log_{10} x - 2 = \log_{10}\left(\frac{x}{k}\right) = \log_{10} x - \log_{10} k$

$2 = \log_{10} k \qquad 10^2 = k$

17. There are two sets of data: the first set (x) has 10 pieces of data and the second set (y) has 20 pieces of data. The mean of the second set is four more than the mean of the first set.

One of the pieces of data from the first set is exchanged with a piece from the second set. As a result the mean of the second set is now one more than the mean of the first set.

'a' - difference between pieces of data

Following this exchange, by how much has the mean of the first set increased?

A 0.5 B 1 C 2 D 4 E 10

$\frac{\Sigma x}{10} + 4 = \frac{\Sigma y}{20}$

$2\Sigma x + 80 = \Sigma y$

$\frac{\Sigma x + a}{10} + 1 = \frac{\Sigma y - a}{20}$

$2\Sigma x + 2a + 20 = \Sigma y - a$

$2\Sigma x + 3a + 20 = 2\Sigma x + 80$

$3a = 60$

$a = 20$

$\frac{\Sigma x + 20}{10} = \frac{\Sigma x}{10} + 2$

18. Find the value of $cos^2 0 + cos^2 2 + cos^2 4 + \ldots + cos^2 86 + cos^2 88 + cos^2 90$

A 0.5 B 1 C 22 D 22.5 E 23

$1 - \cos^2 a = \sin^2 a = \cos^2(90 - a)$

$1 = \cos^2 a + \cos^2(90 - a)$

$\cos^2 0 + \cos^2 2 + \cos^2 4 + \ldots + \cos^2 88 + \cos^2 90$

46 terms 23 pairs adding to 1

19. The constant c is chosen so that the line $y = c$ divides the region between the graphs of

$y = x$ and $y = \sqrt{x}$ into two regions of equal area.

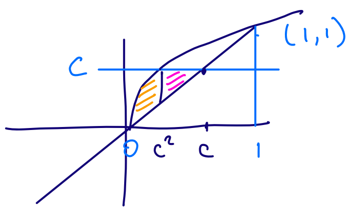
Which of the following equations does c satisfy?

A $6c^4 - 8c^3 + 1 = 0$

B $3c^4 - 4c^3 + 1 = 0$

C $2c^3 - 3c^2 + 1 = 0$

D $4c^3 - 6c^2 + 1 = 0$



$$\int_0^1 x^{1/2} - x \, dx = \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{2}x^2\right]_0^1 = \tfrac{2}{3} - \tfrac{1}{2} = \tfrac{1}{6}$$

$$\int_0^{c^2} x^{1/2} - x \, dx + \tfrac{1}{2}(c - c^2)^2 = \tfrac{1}{12} \quad \left(\tfrac{1}{2} \text{ of } \tfrac{1}{6}\right)$$

$$\left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{2}x^2\right]_0^{c^2} + \tfrac{1}{2}(c^2 - 2c^3 + c^4) = \tfrac{1}{12}$$

$$\tfrac{2}{3}c^3 - \tfrac{1}{2}c^4 + \tfrac{1}{2}c^2 - c^3 + \tfrac{1}{2}c^4 - \tfrac{1}{12} = 0$$

$$-\tfrac{1}{3}c^3 + \tfrac{1}{2}c^2 - \tfrac{1}{12} = 0$$

$\times -12$ $\quad 4c^3 - 6c^2 + 1 = 0$

20. The curve C has equation $y = x^3 - x^4$

The straight line L is a tangent to C at two distinct points. Find the equation of L.

A $y = \dfrac{1}{8}x + \dfrac{1}{64}$

B $y = \dfrac{1}{8}x - \dfrac{1}{64}$

C $y = \dfrac{1}{2}x + \dfrac{1}{32}$

D $y = \dfrac{1}{2}x - \dfrac{1}{32}$

$x^3 - x^4 = mx + c$ Need 2 repeated roots

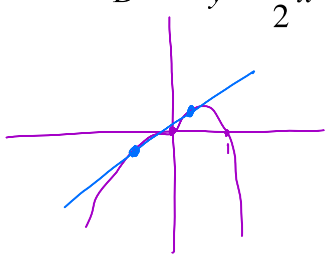
$x^4 - x^3 + mx + c = 0$ $\quad (x-p)^2(x-q)^2 = 0$

$(x^2 - (p+q)x + pq)^2 = 0$

$x^4 - 2(p+q)x^3 + (2pq + (p+q)^2)x^2 - 2pq(p+q)x + p^2q^2 = 0$

$2(p+q) = 1$ $\quad 2pq + \frac{1}{4} = 0$ $\quad m = -2(-\frac{1}{8})(\frac{1}{2}) = \frac{1}{8}$

$p + q = \frac{1}{2}$ $\quad pq = -\frac{1}{8}$ $\quad c = (pq)^2 = \frac{1}{64}$



Mock TMUA Set A: Paper 2

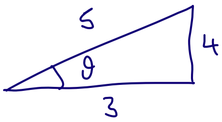
20 questions
75 minutes
No calculator allowed

1. Given that $cos\theta = -0.6$ and $0 < \theta < 180$ find the value of $tan\theta$

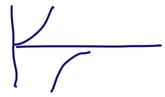
$\cos\theta < 0$

$90 < \theta < 180$

A $\frac{4}{3}$ B $-\frac{4}{3}$ C $-\frac{3}{4}$ D $\frac{4}{5}$ E $-\frac{\sqrt{34}}{3}$



$\tan\theta = -\frac{4}{3}$



$\tan\theta < 0$

2. Find the complete set of values of k for which the line $y = 2x - 1$ crosses or touches the curve $y = x^2 + kx + 3$

A $-2 \le k \le 6$
B $-1 \le k \le 3$
C $-3 \le k \le 3$
D $k \le -2$ or $k \ge 6$
E $k \le -1$ or $k \ge 3$

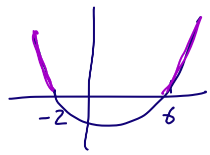
$x^2 + kx + 3 = 2x - 1$

$x^2 + (k-2)x + 4 = 0$

$\Delta \geq 0 \quad (k-2)^2 - 4(4) \geq 0$

$k^2 - 4k - 12 \geq 0$

$(k-6)(k+2) \geq 0$



3. Consider the statement:

No positive integer N that is less than 20 can be written as the sum of two non-prime integers that are greater than 1.

Which of the following is a counterexample to this statement?

I $N = 8$ II $N = 13$ III $N = 24$

$4+4$ $\quad 4+9 \quad$ x not less than 20

A none of them
B I and II only
C II and III only
D I and III only
E I, II and III

4. Five sealed jars, labelled A, B, C, D and E, each contain the same (non-zero) number of £1 coins.

The following statements are attached to the jars:

A This jar contains less than £4 £1 ⇒ C true £2,3 ⇒ E true

B This jar contains £5 or £6 £5 ⇒ D true £6 ⇒ C true

C This jar contains £1 or £6 £1 ⇒ A true £6 ⇒ B true

D This jar contains more than £3 and less than £6 4 5 £5 ⇒ B true

E This jar contains £2 or £3 ⇒ A true

Exactly one of the jars has a true statement attached to it. Which jar is it?

5. A child is chosen at random from a group. Each child is equally likely to be chosen.
Which of the following conditions is/are **necessary** for the probability that the child has a pet to equal exactly $\frac{4}{9}$?

I Exactly 5 children in the group do not have a pet. Sufficient, not necessary

II The number of children in the group is divisible by 4. X

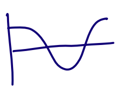
III The number of children in the group with a pet is divisible by 3. X

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

6. What is the smallest positive value of a for which the graph of

$y = cos(\frac{3}{2}x - 60)$ has a line of symmetry at $x = a$?

A 20 B 30 C 40 D 45 E 60



cos x has symmetry at $x = 0, 180$

$\frac{3}{2}x - 60 = 0$

$\frac{3}{2}x = 60$

$x = 40$

7. The real numbers a, b, c and d satisfy $0 < a < b < c < d$

Which of the following inequalities must be true?

$a + c < b + c$

$b + c < b + d$

I $\quad$ $a + c < b + d$ ✓

II $\quad$ $d - c > b - a$ ✗ $\quad$ (10, 9, 8, 1)

III $\quad$ $\dfrac{b}{c} < \dfrac{d}{a}$ $\quad$ $ab < cd$ ✓

A none of them

B I only

C II only

D III only

E I and II only

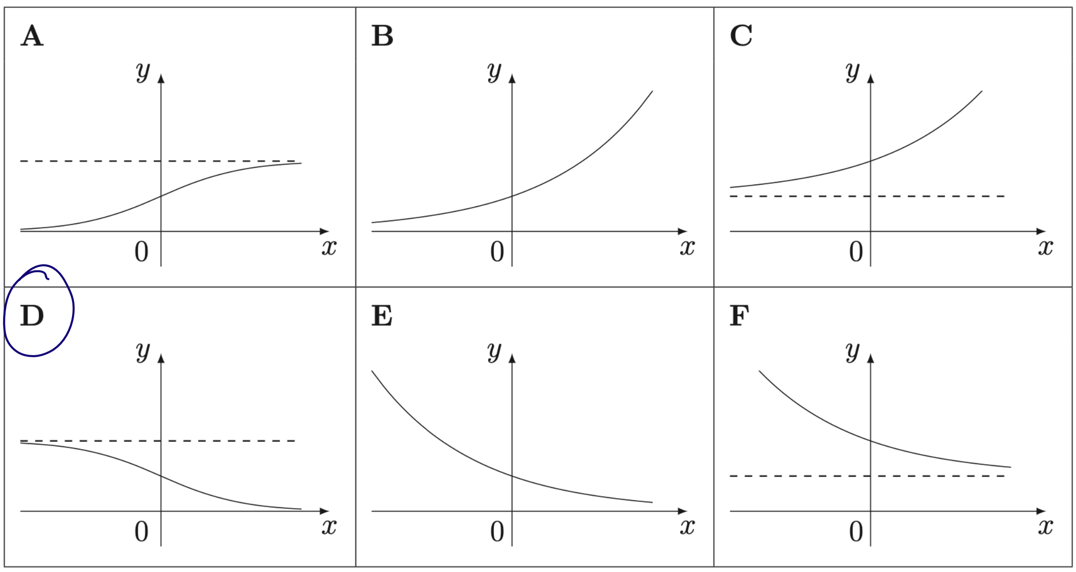
F II and III only

G I and III only

H I, II and III

8. Which one of the following shows the graph of $\qquad$ and $y = \dfrac{3}{2^x + 1}$

(Dotted lines indicate asymptotes.) $\quad (0, \tfrac{3}{2}) \quad x \to \infty \;\; y \to 0 \quad x \to -\infty \;\; y \to 3$



9. How many positive solutions for x are there for the equation:

$$\int_0^x (|t-4|-3)\ dt = 0$$

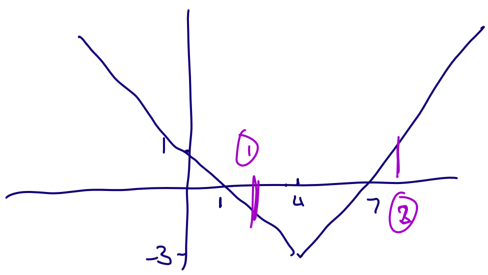
A 0

B 1

C 2

D 4

E infinitely many



2 solutions

10. Consider the following incorrect attempt at a proof, given that a and b are real numbers such that $a = b$. On which line does the first mistake occur?

A $a^2 = ab$

B $a^2 - b^2 = ab - b^2$

C $(a-b)(a+b) = b(a-b)$

D $a + b = b$ or $a = b$

E $2b = b$

F $2 = 1$

11. The function f is defined such that $f(n) = \begin{cases} 0 & \text{if n is a multiple of 5} \\ 1 & \text{if n is not a multiple of 5} \end{cases}$

The function g is defined such that $g(n) = \begin{cases} 0 & \text{if n is a multiple of 7} \\ 1 & \text{if n is not a multiple of 7} \end{cases}$

The function h is defined as $h(n) = (1 - f(n))(1 - g(n))$

What is the value of $h(3) + h(6) + h(9) + \ldots + h(2004) + h(2007)$

A 13

B 19

C 38

D 57

E 152

$h(3) = 0$ $h(6) = 0$

$h(15) = 0$ $h(35) = 1$ (need multiples of 35)

$3 \overline{)2007}$ = 669 → 669 terms in the sum

$35 \overline{)669}$ = 19
35
319
315
4

$19 \times 1 = \boxed{19}$

12. A sequence is defined by $A_n = A_{n-1} + (-1)^{n+1}(n)^2$

What is the value of $A_{31} - A_{29}$

A 30

B 45

C 57

D 61

E 90

$A_{31} - A_{29} = A_{30} + 31^2 - A_{29}$
$= A_{29} - 30^2 + 31^2 - A_{29} = 31^2 - 30^2 = (31-30)(31+30)$
$= 1 \times 61 = \underline{61}$

13. Consider the following two statements about the polynomial $p(x)$ with exactly two stationary points at $x = a$ and $x = b$ where $a < b$

P: $p(x)$ has a maximum at $x = a$

Q: $p'(x) < 0$ for $a < x < b$

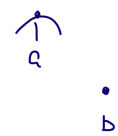
Which of the following is correct?

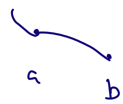
A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q

P $\Rightarrow$  $\Rightarrow$ Q   P sufficient

Q $\Rightarrow$  $\not\Rightarrow$ P   P not necessary

14. Find the minimum value of $f(x) = \dfrac{cosx + 4}{9 + 6cosx - sin^2x}$ for $x \in \mathbb{R}$

A 1

B 3

C $\frac{1}{9}$

D $\frac{1}{3}$

E $\frac{4}{9}$

$9 + 6\cos x - 1 + \cos^2 x$

$\cos^2 x + 6\cos x + 8$

$(\cos x + 4)(\cos x + 2)$

$f(x) = \dfrac{1}{\cos x + 2}$

Min $f(x)$ when max $\cos x + 2$

15. The function $f(x)$ is such that

$$\int_0^3 f(x)\ dx = 8 \qquad \int_{-2}^3 f(x)\ dx = 11 \qquad \int_{-3}^5 f(-x)\ dx = 2$$

Find $\int_{-5}^{-2} f(x)\ dx$

A - 9
B - 6
C - 3
D 3
E 6

$\int_0^3 = 8$ $\quad \int_{-2}^3 = 11$ $\quad \int_{-3}^5 f(-x) = 2$

$\int_{-2}^0 = \overset{11-8}{3}$ $\quad \int_{-5}^3 f(x) = 2$

$\int_{-5}^0 = \overset{2-8}{-6}$ $\quad \int_{-5}^{-2} = -6-3 = -9$

16. $f(x) = x^3 + (a+2)x^2 - 2x - b$ where a and b are positive constants.

Given that $(x-2)$ and $(x+a)$ are factors of f(x), find the value of a - b.

A - 21
B - 8
C 3
D 8
E 21

$f(2) = f(-a) = 0$

$8 + 4(a+2) - 4 - b = 0$

$4a - b + 12 = 0 \qquad b = 4a + 12$

$-a^3 + a^2(a+2) + 2a - b = 0$

$2a^2 + 2a - b = 0$

$2a^2 + 2a - 4a - 12 = 0$

$a^2 - a - 6 = 0$

$(a-3)(a+2) = 0 \qquad a = 3$

$b = 24$

$a - b = -21$

17. Which one of the following is a **necessary and sufficient** condition for the line with equation $y = mx + c$ to be a tangent to the circle with equation $x^2 + y^2 = a^2$

A $m = 0$ and $c = a$ S not N
B $m = 0$ and $c = \pm a$ S not N
C $m^2 + c^2 = a^2$
D $c^2 = \dfrac{a^2}{m^2}$
E $c^2 = a^2(m^2 + 1)$

$x^2 + m^2x^2 + 2mcx + c^2 - a^2 = 0$

$\Delta = 0$

$4m^2c^2 - 4(1+m^2)(c^2 - a^2) = 0$

$m^2c^2 - (c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$-c^2 + a^2 + m^2a^2 = 0$

$c^2 = a^2(m^2+1)$

18. Let $a, b, c > 0$. The equations: $log_a b = c$ $a = b^c$

$a^c = b$ $a^{c^2} = b^c = a$

$(a^c)^c = b^c$

A specify a b and c uniquely

B specify c uniquely but have infinitely many solutions for a and b

C specify a and b uniquely but have infinitely many solutions for c

D have no solutions for a, b and c

$c^2 = 1$ $a = b$

$c = 1$

19. The two circles with equations below touch each other externally.
$x^2 + y^2 - 4x + 12y + 4 = 0$ and $x^2 + y^2 = a^2$

Find the value of a

$(x-2)^2 + (y+6)^2 - 4 - 36 + 4 = 0$

$(x-2)^2 + (y+6)^2 = 6^2$

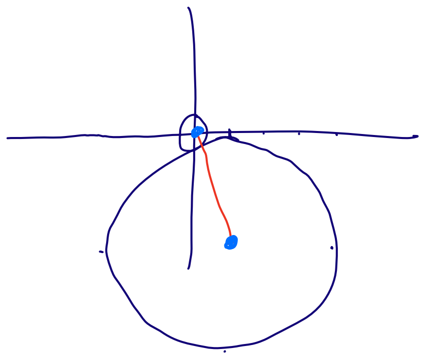
A 6

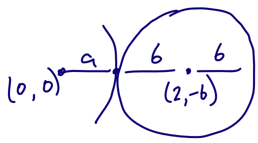
B $4\sqrt{2}$

C $2\sqrt{10} - 6$

D $2\sqrt{10} + 6$

E $4\sqrt{2} + 6$





$a + 6 = \sqrt{40}$

$a = 2\sqrt{10} - 6$

20. Let x be a real number, and consider the inequality $x^2 + 1 \geq 10$
Which of the following conditions on x is **necessary but not sufficient** for this to be true?

A $x \leq -3$ or $x \geq 3$ N + S

B $x \neq 0$

C $x > 10$ S not N

D $x = 3$ S not N

E $x > 0$ not N