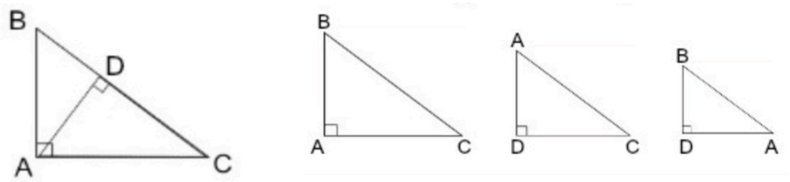
TMUA Practice - Geometry - Notes

This worksheet features questions that have some geometry component. Often some trigonometry is required (Pythagoras, sine rule or cosine rule), but some questions can also be solved using simpler techniques such as the properties of similar shapes. Look out for equal lengths and angles, hidden right angles, similar triangles, circle theorems and any symmetry.

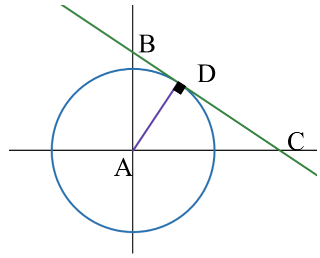
In particular note the following properties of quadrilaterals and the triangles formed by their diagonals:

Rectangle	diagonals equal and bisect each other
Parallelogram	diagonals bisect each other (2 pairs of congruent triangles)
Rhombus	diagonals bisect each other at right angles (4 congruent triangles)

It is often hard to spot similar triangles when one is rotated or reflected. In particular if a perpendicular line is drawn inside a right angled triangle (see diagram below), then two similar triangles are formed inside.



Note how this method also applies when for example we are finding the shortest distance from a tangent to the origin.Depending on the information given (gradients, intercepts etc) it is often possible to find this distance without needing to find the coordinates of point D.



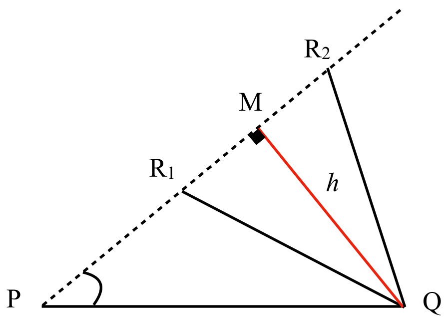
One topic that is very different to standard A Level questions involves the ambiguous case where information about a triangle is given in the form SSA (which does not prove congruence unlike SAS etc). My top tip is always to draw a diagram in the same orientation as follows: if information is given about angle P, side PQ and side QR, then start with the angle on the left (with a dotted line to show we do not know the length of PR), and side PQ at the bottom. At this point you should draw in the perpendicular from Q to the opposite side (which has a fixed length h). Then you need to compare the length of the final side RQ with h. Depending on the length of RQ there could be 0, 1 or 2 possible triangles. Think of QR as a fixed length rotating about the point Q - wherever it crosses the dotted line is the position of R.

If $RQ < h$ the third side does not reach PR and there are no triangles

If $RQ = h$ we can draw one distinct right angled triangle (PQM)

If $h < RQ < PQ$ there are two possible positions for R and hence two possible triangles - this is the ambiguous case.

If $RQ > PQ$ we can only draw one triangle (PQR_2).



Note also that R_1R_2Q is an isosceles triangle so we also have $MR_1=MR_2$ and the right angled triangles MPQ and MQR can also be helpful. By calculating the areas of PQR_1 and PQR_2 using $\frac{1}{2}PR \times PQ \times sinP$ we can also see that the ratio of the areas is the same as the ratio $PR_1 : PR_2$

Finally questions on 3D shapes will generally involve some trigonometry. Note that finding the shortest distance on the outside of a 3D shape is most easily done by opening the shape into a 2D net and considering the straight line distance between the two points. Just be careful that there is often more than one way to travel along the outside of the shape.

TMUA style questions - Geometry

1. A regular octagon with side length 1 is drawn inside a circle so that each vertex of the octagon lies on the perimeter of the circle.

   What is the area of the circle?

   A $\frac{(2-\sqrt{2})\pi}{2}$ B $\frac{(1+\sqrt{2})\pi}{2}$ C $\sqrt{2}\pi$ D $\frac{(2+\sqrt{2})\pi}{2}$ E 2π

2. An equilateral triangle with side length 6cm is inscribed in a circle (the circle passes through each of the vertices of the triangle).

   Which of the following is an expression for the circumference of the circle, in cm.

   A 6π B $4\pi\sqrt{3}$ C $\frac{10\pi\sqrt{3}}{3}$ D $3\pi\sqrt{3}$ E $\frac{4\pi\sqrt{3}}{3}$

3. A sector of a circle of radius x cm contains an angle of θ radians.

The perimeter of the sector is 12 cm.

What is the maximum possible value of the area of the sector as x varies?

A 3 B 6 C 9 D 12 E 18

4. Triangle ABC is right angled at B. The perpendicular from B to the hypotenuse meets AC at D.

Given that the ratio of AD:CD is 1:4, which fraction best represents the following

$$\frac{\text{length of hypotenuse AC}}{\text{sum of lengths of AB and BC}} \ ?$$

A $\frac{2\sqrt{3}}{5}$ B $\frac{\sqrt{5}}{3}$ C $\frac{4}{5}$ D $\frac{\sqrt{3}}{2}$ E $\frac{2\sqrt{5}}{5}$

5. Given that $x^2 + y^2 = 1$, what is the greatest possible value of $x - 3y$

A $3\sqrt{2}$

B $\sqrt{10}$

C 3

D $2\sqrt{2}$

E 1

6. Circle C_1 has equation $(x + 3)^2 + (y - 3)^2 = 5$

Circle C_2 has equation $(x - 9)^2 + (y - 3)^2 = 20$

The straight line L passes between these circles with a positive gradient and is a tangent to both circles. The angle between L and the x-axis is θ. Find $cos\theta$

A $\frac{\sqrt{5}}{6}$ B $\frac{\sqrt{5}}{4}$ C $\frac{2}{3}$ D $\frac{\sqrt{11}}{4}$ E $\frac{\sqrt{3}}{2}$

7. A triangle ABC is to be drawn with the following measurements.

$AB = 8$ cm, angle $BAC = 30°$, $BC = x$ cm.

For which values of x can a unique triangle ABC be drawn ?

A $x = 4$ or $x \geq 8$

B $4 \leq x \leq 8$

C $x \geq 4$

D $x \geq 8$

E $0 \leq x \leq 4$ or $x = 8$

8. A triangle ABC is to be drawn with the following measurements.

$AC = b$ $BC = 2a$ angle $ABC = 45°$ where a and b are constants

For which value of b can two distinct triangles ABC be drawn ?

A $b = \dfrac{\sqrt{2}}{2}\, a$

B $b = a$

C $b = \sqrt{2}\, a$

D $b = \dfrac{3}{2}\, a$

E $b = 2a$

9. A triangle ABC is to be drawn with the following measurements.

$AB = x$ $\quad\quad$ $BC = 6$ $\quad\quad$ angle $BAC = 60°$ $\quad\quad$ angle $ABC > 90°$

Find the range of values for x such that only one possible triangle can be drawn, satisfying the conditions above.

A $\quad 0 < x < 2\sqrt{3}$

B $\quad 2\sqrt{3} < x < 6$

C $\quad 2\sqrt{3} < x \leq 4\sqrt{3}$

D $\quad 0 < x < 6$

E $\quad 6 < x \leq 4\sqrt{3}$

10. A triangle ABC is to be drawn with the following measurements.

$AB = \sqrt{13}\ a$ $\quad\quad$ $BC = \sqrt{10}\ a$ $\quad\quad$ angle $BAC = \theta°$ $\quad\quad$ where $tan\theta = \dfrac{3}{2}$

Given that two possible triangles can be drawn, what is ratio of the area of the larger triangle to the area of the smaller triangle?

A $\quad \sqrt{13} : 3$

B $\quad \sqrt{13} : \sqrt{10}$

C $\quad 3:1$

D $\quad \sqrt{10} : 1$

E $\quad 13 : 3$

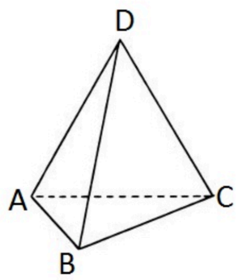
11. Consider a triangle ABC with the following measurements.

$AB = 3a$ $\quad$ $BC = k$ $\quad$ angle $BAC = 45°$

Given that two possible triangles can be drawn, find the value of k that means the area of the larger triangle is double the area of the smaller one.

A $2\,a$ $\quad$ B $\sqrt{5}\,a$ $\quad$ C $3\,a$ $\quad$ D $3\sqrt{2}\,a$ $\quad$ E $6\,a$

12. The diagram shows a tetrahedron with base ABC and vertex D. All the edges of the tetrahedron are the same length. The base ABC is stuck to a table.



M is the point that lies on AB such that the ratio of AM:BM is 1:3.

Find the ratio of MC:BC where MC is the shortest distance from M to C along the **upper** outer surfaces of the tetrahedron (not across the base).

A $\sqrt{3} : 2$ $\quad$ B $\sqrt{13} : 4$ $\quad$ C $\sqrt{13} : 3$ $\quad$ D $\sqrt{37} : 4$ $\quad$ E $\sqrt{21} : 4$

13. A right cone has height $5\sqrt{5}$ cm and slant height 15 cm.

Points A and B lie on the circumference of the base and are diametrically opposite.

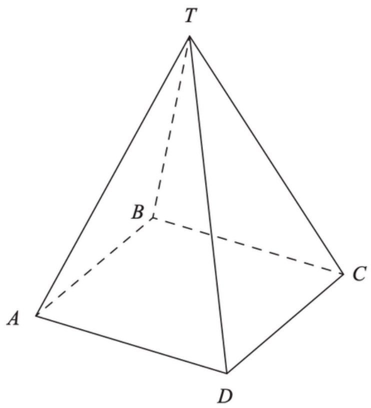
What is the shortest distance along the curved surface from A to B?

A $15\sqrt{3}$ B $5\sqrt{3}\pi$ C 30 D 10π E $5\sqrt{5}\pi$

14. A square based pyramid has base ABCD and vertex T. The base edges have length 6m, and the sloping edges (AT, BT, CT, DT) have length 5m.

Point P is on AT such that PT is 3m.
Point Q is on CT such that QT is 3m.

Find the shortest distance, in metres, along the outer surface of the pyramid from P to Q.



A $\frac{36}{5}$ B 6 C $\frac{35}{6}$ D $\frac{144}{25}$ E $\frac{27}{5}$