MM4. Trigonometry Worksheet - introducing key skills needed for the TMUA.

Q1 / Q2: solve equations using your knowledge of exact values and simple trigonometric identities; find all solutions for a given domain; spot disguised quadratics; work in degrees and radians.

Q3: sketch graphs to count the number of solutions to an equation without solving it.

Q4: analyse trigonometric functions to determine where their product or quotient is positive or negative, or where one function is greater than another. You may have to identify the actual interval, or calculate what fraction of the interval is required.

Q5 / Q6: perform successive transformations to find the new equation of a transformed curve, or the new position of particular coordinates.

Q7: find maximum or minimum values of expressions involving trigonometric functions, especially by considering the range of sine and cosine.

Q8: solve questions requiring the sine and cosine rule, often with lengths or angles given in terms of algebraic expressions

Q9: solve questions involving the ambiguous case for triangles (angle-side-side).

Q10: find the shortest path between 2 points over the surface of a 3D shape by considering the possible paths, and then drawing a net of the relevant part of the 3D shape that represents these paths as straight lines.

1. Solve the following trigonometric equations in the range given

a) $\cos(2\theta + 30) = 0.5 \quad 0 \leq \theta \leq 360°$

b) $\tan(5\theta - 35) = \sqrt{3} \quad 0 \leq \theta \leq 90°$

c) $2\sin^2\theta + 3\cos\theta = 0 \quad 0 \leq \theta \leq 360°$

d) $\sin(3\theta + 72) = \cos 48 \quad 0 \leq \theta \leq 180°$

e) $\dfrac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta \quad 0 \leq \theta \leq 360°$

f) $\cos^2(2x) + \sqrt{3}\sin(2x) - \dfrac{7}{4} = 0 \quad 0 \leq x \leq 360°$

2. Solve, in radians, the following trigonometric equations, giving your answers in terms of π .

a) $\cos 2x = \cos \frac{2\pi}{5} \quad 0 \leq x \leq 2\pi$

b) $8 \sin(\frac{\pi}{3} - 2x) = 4 \quad 0 \leq x \leq 2\pi$

c) $\sin^2 \frac{3\theta}{2} = \frac{1}{2} \quad 0 \leq \theta \leq 2\pi$

d) $2 \sin^2 y - 5 \cos y + 1 = 0 \quad 0 \leq y \leq 2\pi$

e) $\tan^4 x - \tan^2 x = 6 \quad 0 \leq x \leq 2\pi$

3. Find the number of solutions to the following equations:

a) $x \tan x = 2$ $\quad -2\pi \leq x \leq 2\pi$

b) $x \sin 3x + \cos 3x = 0$ $\quad 0 \leq x \leq 2\pi$

c) $2x + \pi \sin x = 3\pi$

d) $(x+3)(3-x) = 9\cos\frac{\pi}{2}x$

e) $2^x + 2\sin(\pi x) = 3$

4. Find the complete set of values for which:

a) $y \geq 0$ where $y = \tan x \cos 2x$ $0 \leq x \leq \pi$

b) $y \geq 0$ where $y = (1 - 2\sin x)\cos x$ $0 \leq x \leq \pi$

c) $y \leq 0$ where $y = (1 + \cos 2x)\cos 2x$ $0 \leq x \leq \pi$

d) $2\cos 2x \leq \frac{4x}{\pi} - 1$ $0 \leq x \leq 2\pi$

e) $\tan x \leq \sin 2x$ $0 \leq x \leq \pi$

f) $\sin 2x > \sin x$ $0 \leq x \leq 2\pi$

Find the fraction of the interval $0 \leq x \leq 2\pi$ for which this inequality is true.

5. Find the equation of the new curves after the following transformations:

a) The curve $y = \cos x$ is stretched in the horizontal direction by a scale factor of $\frac{1}{2}$, and the resulting curve is translated by 4 units in the positive y-direction.

b) The curve $y = \sin x$ is stretched in the vertical direction by a scale factor of 2, followed by a translation by $\frac{\pi}{3}$ units in the positive x-direction.

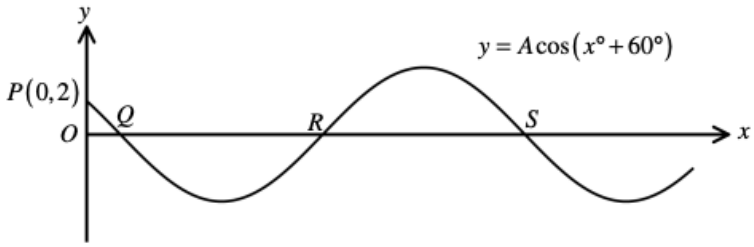
c) The curve $y = \tan x$ is stretched in the horizontal direction by a scale factor of 2, followed by a reflection in the x-axis, followed by a translation by 3 units in the positive y-direction.

d) The curve $y = \sin x$ is reflected in the y-axis, translated by 2 units in the negative y-direction and then translated by $\frac{\pi}{4}$ units in the positive x-direction.

e) The curve $y = \cos x$ is reflected in the line $y = 1$

f) The curve $y = \sin x$ is reflected in the line $y = 2$ followed by a translation by $\frac{\pi}{3}$ units in the negative x-direction.

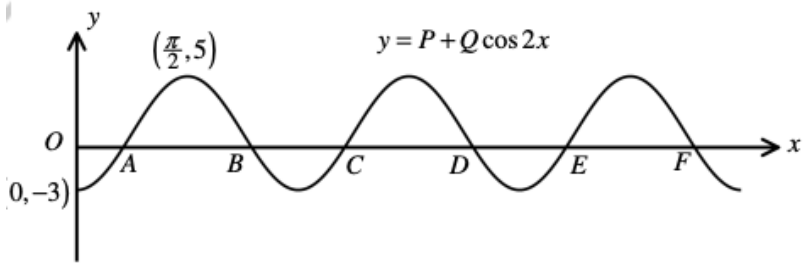
6a) The figure shows part of the graph of the curve with equation $y = A\cos(x + 60)$



The point P (0,2) lies on the curve. Find the value of A and the coordinates of Q, R, and S.

Find the coordinates of the points where the straight line with equation $y = -2\sqrt{3}$ meets this graph.

6b) The figure shows part of the graph of the curve with equation $y = P + Q\cos 2x \qquad x \geq 0$



The points (0 , -3) and ($\frac{\pi}{2}$, 5) lie on the curve. Find the value of P and Q.

Find the coordinates of the points where the straight line with equation $y = 3$ meets this graph for $0 \leq x \leq 2\pi$.

6c) A curve has equation $y = A + k \sin x$, $0 \leq x \leq 2\pi$, where A and k are non-zero constants.

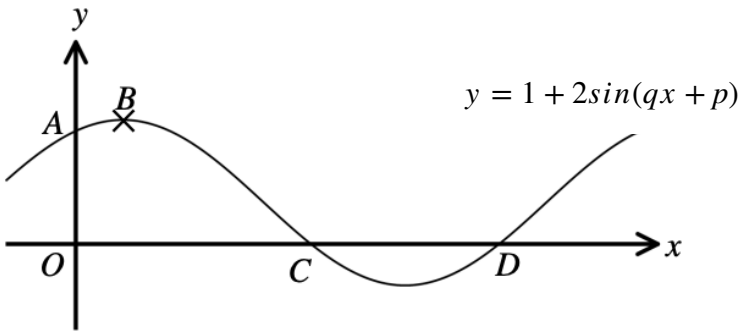
Given that the curve passes through $(\frac{\pi}{6}, 1)$ and $(\frac{7\pi}{6}, 5)$, find the minimum and maximum value of y.

6d) The figure shows part of the graph of $y = 1 + 2\sin(qx + p)$,

where q and p are positive constants, with $0 < p < 90$, $0 < q < 5$

The graph crosses the y-axis at the point A $(0, 1 + \sqrt{3})$ and the x-axis at the points C (50, 0) and D, and point B is a maximum point on the curve.

Find the values of p and q, and the coordinates of B and D.



7.

a) Find the greatest value of the function $f(x) = (2\cos^2(4x + 9) - 5)^2$ for $x \in \mathbb{R}$

b) Find the largest value achieved by $f(x) = 3\cos^2 x + 2\sin x + 1$ for $x \in \mathbb{R}$

c) Find the largest value of $f(x) = (4\sin^2 x + 4\cos x + 1)^2$ for $x \in \mathbb{R}$

d) Find the minimum value of the function $f(x) = 9\cos^4 x - 12\cos^2 x + 7$ for $x \in \mathbb{R}$

e) Find the maximum value of $9^{\sin x} - 4(3^{\sin x}) + \frac{13}{3}$ for $x \in \mathbb{R}$

f) Find the minimum value of $f(x) = \frac{\cos x + 4}{9 + 6\cos x - \sin^2 x}$ for $x \in \mathbb{R}$

8a) In triangle PQR, PR has length $(x^2 + 2x + 2)$, PQ has length $(4\sqrt{2}x + 5\sqrt{2})$, angle PRQ is 45°, and angle PQR is 30°. Find the sum of all possible values of x.

8b) A triangle ABC has $AB = x + 1$, $BC = (5 - x)(x + 1)$, and angle $ABC = 30°$. What is the greatest area of the triangle?

8c) A triangle ABC has $AB = x$, $BC = x + 2$, and $AC = 2x + 1$. For which values of x is angle ABC obtuse?

8d) The lengths of the sides QR, RP and PQ in triangle PQR are $s + 1$, s, and $\sqrt{3}s$. Find the full range of values of s that make angle PRQ an obtuse angle.

9a) In triangle ABC, $AB = 8$, $BC = k$, angle $BAC = 30°$.

For which values of k are there exactly two non-congruent triangles satisfying these conditions.

9b) In triangle ABC, $AB = 6$, $BC = 3\sqrt{3}$, angle $BAC = 45°$.

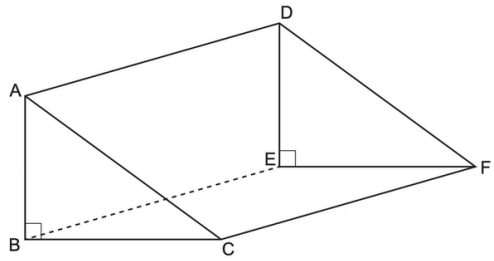
Two non-congruent triangles satisfy these conditions. What is the ratio of their areas?

9c) A triangle ABC has $AB = 10$, angle $BAC = 30°$ and $BC = k$ cm, where k is a constant.

Of the two possible triangles that could be drawn, the larger triangle has side AC twice as long as side AC in the smaller triangle. Find the value of k.

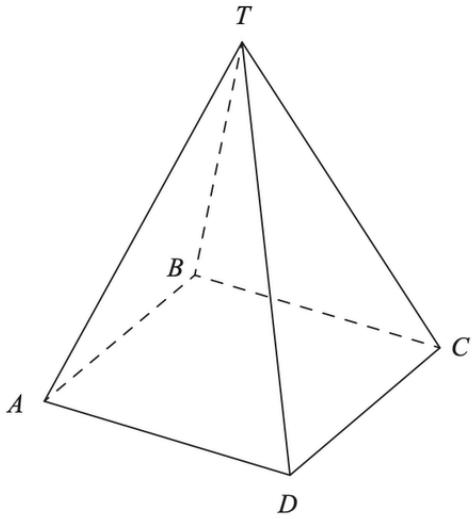
10a) A solid right angled triangular prism, with base *BEFC*, has $AB = BC = 2$m, and $AD = 5$m.

Find the shortest distance, in metres, along the outside of the prism from *B* to the midpoint of *DF*.



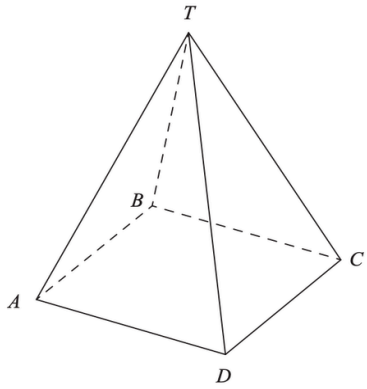
10b) A square based pyramid, with base *ABCD*, and vertex *T* has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the pyramid from the midpoint of *AB* to the midpoint of *CT*.



10c) A square based pyramid has base *ABCD*, where all the sides of the square are 12cm in length. The diagonals of the square intersect at *O*, and the vertex of the pyramid is at *T*, directly above *O*. Each of the sloping edges of the pyramid makes an angle of 60° with the base.

The angle between the face *CDT* and the base *ABCD* is θ. Find $\tan\theta$.



10d) A triangular-based pyramid *PQRS* has horizontal base *PQR* where

$PQ = PR = 8\text{m}$ and $RQ = 12\text{m}$.

The vertex of the pyramid *S* lies directly above the level of *PQR* so that

$SQ = SR = 10\text{m}$ and $SP = 8\text{m}$.

Find the shortest distance of *S* from the base *PQR*.