

TMUA Multiple Choice Practice - Differentiation

1. The gradient of the curve $y = \frac{(4x + \sqrt{x})(x^2 - 3)}{3\sqrt{x}}$ at the point where $x = 1$ is

- A $\frac{1}{3}$ (B) 2 C $\frac{10}{3}$ D 4 E $\frac{20}{3}$

$$y = \frac{4x^3 + x^{5/2} - 12x - 3x^{1/2}}{3x^{1/2}} = \frac{4}{3}x^{5/2} + \frac{1}{3}x^2 - 4x^{1/2} - 1$$

$$\frac{dy}{dx} = \frac{10}{3}x^{3/2} + \frac{2}{3}x - 2x^{-1/2}$$

$$= \frac{10}{3}x^{3/2} + \frac{2}{3}x - \frac{2}{\sqrt{x}}$$

At $x = 1$ $\frac{dy}{dx} = \frac{10}{3} + \frac{2}{3} - 2 = 2$

2. A curve C has equation $y = \frac{x^2 - 2}{\sqrt{x}}$. Find the gradient of C at the point $(2, \sqrt{2})$.

- A $\sqrt{2}$ (B) $\frac{7}{4}\sqrt{2}$ C $\frac{7}{2}\sqrt{2}$ D $4\sqrt{2}$ E $\frac{9}{2}\sqrt{2}$

$$y = x^{3/2} - 2x^{-1/2}$$

$$y' = \frac{3}{2}x^{1/2} + x^{-3/2}$$

At $x = 2$ $y' = \frac{3}{2}\sqrt{2} + \frac{1}{2\sqrt{2}}$

$$= \frac{3}{2}\sqrt{2} + \frac{1}{4}\sqrt{2} = \frac{7}{4}\sqrt{2}$$

3. The curve $y = x^3 + 3\sqrt{5}px^2 + 3px + 13$ has two distinct turning points.

Find the possible values of p .

- (A) $p < 0, p > 0.2$
 B $p \leq 0, p \geq 0.2$
 C $0 < p < 0.2$
 D $0 \leq p \leq 0.2$
 E $p < 0, p > 1.2$
 F $p \leq 0, p \geq 1.2$

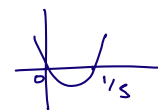
$$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}px + 3p = 0$$

$$\Delta > 0 \quad 180p^2 - 4(3)(3p) > 0$$

$$5p^2 - p > 0$$

$$p(5p - 1) > 0$$

$$p < 0 \quad p > 0.2$$



4. Find the complete set of values of k for which the graph $y = x^3 - 2kx^2 + 4x - k$ has two distinct real stationary points.

A $-3 < k < 3$
 B $k < -3$ or $k > 3$
 C $-\sqrt{3} < k < \sqrt{3}$
D $k < -\sqrt{3}$ or $k > \sqrt{3}$
 E all values of k

$\frac{dy}{dx} = 3x^2 - 4kx + 4$
 $\Delta > 0 \quad 16k^2 - 4(3)(4) > 0$
 $k^2 - 3 > 0$
 $(k - \sqrt{3})(k + \sqrt{3}) > 0$
 $k < -\sqrt{3}, k > \sqrt{3}$

5. Given that the cubic equation $f(x) = p^{\frac{2}{3}}x^3 + px^2 + p^{\frac{1}{3}}x + 3$ where p is a positive constant has exactly one point where $f'(x) = 0$, find the value of p .

- A $\frac{1}{4}$ B $\frac{3}{4}$ C 1 **D** 3 E 6

$f'(x) = 3p^{\frac{2}{3}}x^2 + 2px + p^{\frac{1}{3}} = 0$
 $\Delta = 0 \quad 4p^2 - 4(3p^{\frac{2}{3}})(p^{\frac{1}{3}}) = 0$
 $p^2 - 3p = 0$
 $p(p - 3) = 0$
 $p > 0 \quad p = 3$

6. Consider the function given by $f(x) = x^{\frac{1}{5}}(x^2 - 2x + 1)$
 The fraction of the interval $0 < x < 2$ for which $f(x)$ is decreasing is

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{3}{5}$ D $\frac{5}{6}$ E $\frac{10}{11}$

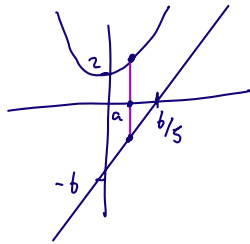
$f(x) = x^{\frac{1}{5}} - 2x^{\frac{6}{5}} + x^{\frac{1}{5}}$
 $f'(x) = \frac{11}{5}x^{\frac{6}{5}} - \frac{12}{5}x^{\frac{1}{5}} + \frac{1}{5}x^{-\frac{4}{5}} < 0$
 $x^{-\frac{4}{5}}(11x^2 - 12x + 1) < 0$
 $x^{-\frac{4}{5}}(11x - 1)(x - 1) < 0$
 $\frac{1}{11} < x < 1$

fraction = $\frac{1 - \frac{1}{11}}{2} = \frac{5}{11}$

> 0 for $0 < x < \frac{1}{11}$

7. A curve has equation $y = 3x^2 + 2$ and a line has equation $y = 5x - 6$.
What is the shortest distance parallel to the y-axis between the curve and the line?

- A $\frac{5}{6}$ B $\frac{6}{5}$ C $\frac{49}{12}$ **D** $\frac{71}{12}$ E 8



$$(a, 3a^2 + 2) \quad (a, 5a - 6)$$

$$D = 3a^2 + 2 - 5a + 6$$

$$= 3a^2 - 5a + 8$$

$$\frac{dD}{da} = 6a - 5 = 0$$

$$a = \frac{5}{6}$$

$$D = 3\left(\frac{25}{36}\right) - \frac{25}{6} + 8$$

$$= \frac{25}{12} - \frac{50}{12} + \frac{96}{12} = \frac{71}{12}$$

8. A curve C has equation $y = 2x^3 - 5x^2 + a$ where a is a constant.

The tangent to C at $x = 2$ and the normal to C at $x = 1$ meet on the x-axis.

The value of a is

- A $\frac{1}{4}$ B $\frac{2}{3}$ C 4 D 6 **E** $\frac{8}{3}$

$$y' = 6x^2 - 10x$$

at $x = 2$ $y' = 4$ $x = 1$ $y' = -4$

$$x = 2 \quad y = 16 - 20 + a = a - 4$$

$$x = 1 \quad y = a - 3$$

Tangent $m = 4$ $(2, a - 4)$

$$y - (a - 4) = 4(x - 2)$$

$$4 - a = 4x - 8$$

$$12 - a = 4x$$

$$40 - 15a = 0$$

$$a = \frac{40}{15} = \frac{8}{3}$$

Normal $m = \frac{1}{4}$ $(1, a - 3)$

$$y - (a - 3) = \frac{1}{4}(x - 1)$$

$$3 - a = \frac{1}{4}x - \frac{1}{4}$$

$$\frac{13}{4} - a = \frac{1}{4}x$$

$$52 - 16a = 4x$$

9. The point P lies on the curve with equation $y = x^2$ so that its distance from the point $Q(-5, -1)$ is least. Find the distance PQ .

- A** $2\sqrt{5}$ B $\sqrt{26}$ C $4\sqrt{5}$ D 20 E 26

$$(-5, -1) \quad (x, x^2)$$

$$D^2 = (x + 5)^2 + (x^2 + 1)^2$$

$$= x^2 + 10x + 25 + x^4 + 2x^2 + 1$$

$$= x^4 + 3x^2 + 10x + 26$$

$$\frac{dD}{dx} = 4x^3 + 6x + 10 = 0$$

$$2x^3 + 3x + 5 = 0$$

$$(x + 1)(2x^2 - 2x + 5) = 0$$

$$(x + 1) \left(\Delta < 0 \right)$$

$$x = -1 \quad \text{no solutions}$$

$$D^2 = 4^2 + 4 = 20$$

$$D = \sqrt{20} = 2\sqrt{5}$$

10. How many real roots does the equation $y = 3x^4 - 16x^3 + 18x^2 - 5$ have?

- A 1 B 2 **C 3** D 4 E 5

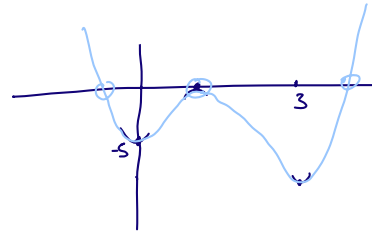
$$\frac{dy}{dx} = 12x^3 - 48x^2 + 36x = 0$$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0$$

$$x(x-1)(x-3) = 0$$

W
+ve graph



$x=0$ $y=-5$
 $x=1$ $y=3-16+18-5=0$
 $x=3$ $y=243-16(27)+18(9)-5 < 0$

$$\frac{d^2y}{dx^2} = 36x^2 - 96x + 36$$

> 0 $x=0$ min
 < 0 $x=1$ max
 > 0 $x=3$ min

11. What is the highest term in x of the following polynomial

$$\frac{d^2}{dx^2} [(x^6 + 2)^2(x^4 - 3)^4] - \frac{d}{dx} [(3x^5 - 1)^3(x^2 + 4)^6]$$

- A $26x^{25}$ **B** $27x^{26}$ C $28x^{26}$ D x^{27} E $28x^{28}$

$$(x^6)^2 (x^4)^4 = x^{28}$$

$12+16$
 $28x^{27}$
 $28 \cdot 27 x^{26}$

$$(3x^5)^3 (x^2)^6$$

15 12
 $27x^{15} x^{12}$
 $27x^{27}$
 $27^2 x^{26}$

$$27(28-27)x^{26}$$

$27x^{26}$

12. A water tank, with volume $500m^3$, is to be made in the shape of a cuboid with a square base and no top. What is the least amount of metal in m^2 required to make this tank?

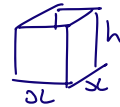
- A $100\sqrt{2}$
 B $100 + 50\sqrt{2}$
 C 200
D 300
 E $50 + 200\sqrt{2}$

$$V = 500$$

$$x^2 h = 500$$

$$SA = x^2 + 4xh$$

$$= x^2 + \frac{2000}{x}$$



$$\frac{dSA}{dx} = 2x - \frac{2000}{x^2} = 0$$

$$x^3 = 1000$$

$$x = 10$$

$$h = 5$$

$$SA = 100 + 200 = 300$$

13. A curve C has equation given by $f(x) = 2p^3 + 3p^2x - 2px^2 + x^3$ where p is real.

The gradient of the normal to C at $x = 1$ is M .

What is the least possible value of M as p varies?

- A $-\frac{7}{2}$ B $-\frac{5}{2}$ C $-\frac{5}{3}$ **D** $-\frac{3}{5}$ E $\frac{2}{3}$

$$f'(x) = 0 + 3p^2 - 4px + 3x^2$$

$$f'(1) = 3p^2 - 4p + 3 \quad \text{- grad of tang.}$$

$$M = \frac{1}{-3p^2 + 4p - 3} \quad \text{least } M \text{ when max } (-3p^2 + 4p - 3)$$

$$-3\left(p^2 - \frac{4}{3}p + 1\right)$$

$$-3\left[\left(p - \frac{2}{3}\right)^2 + \frac{5}{9}\right]$$

$$-3\left(p - \frac{2}{3}\right)^2 - \frac{5}{3}$$

$$\text{Max} = -\frac{5}{3} \quad \text{so } M = -\frac{3}{5}$$

or $\frac{d}{dp} = -6p + 4 = 0$
 $p = \frac{2}{3}$
 $-3\left(\frac{4}{9}\right) + \frac{8}{3} - \frac{9}{3} = -\frac{5}{3}$

14. How many real roots does the equation $y = x^5 + 5x^3 - 20x + 14$ have?

- A 1 **B** 2 C 3 D 4 E 5

$$\frac{dy}{dx} = 5x^4 + 15x^2 - 20 = 0$$

$$x^4 + 3x^2 - 4 = 0$$

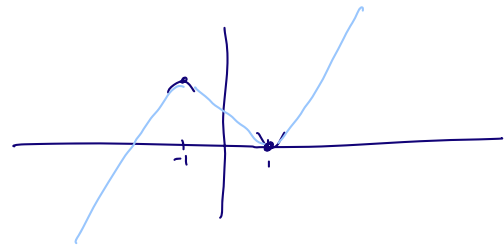
$$(x^2 - 1)(x^2 + 4) = 0$$

$$x = \pm 1 \quad \text{no solns}$$

$$x = 1 \quad y = 1 + 5 - 20 + 14 = 0 \quad \text{min}$$

$$x = -1 \quad y = -1 - 5 + 20 + 14 > 0 \quad \text{max}$$

$$\frac{d^2y}{dx^2} = 20x^3 + 30x = 10x(2x^2 + 3)$$



15. A cubic curve has equation $y = ax^3 + bx^2 + cx + d$ where a, b, c, d are non-zero constants.

Given that this curve has one local maximum and one local minimum, which of the following statements is necessarily true:

A $b^2 > 3ac$

B $b^2 > 4ac$

C $c^2 > 4bd$

D If $a > 0$, then $d > 0$

E If $a > 0$, then $d < 0$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c = 0$$

2 distinct solutions $\Delta > 0$

$$4b^2 - 4(3a)c > 0$$

$$b^2 - 3ac > 0$$

16. The function $f(x) = \frac{3x-2}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$

The complete set of values of x for which the function is decreasing is

- A $x < -\frac{4}{3}, x > 0$
- B** $-\frac{4}{3} < x < 0$
- C $-\frac{4}{3} < x < \frac{4}{3}, x \neq 0$
- D $-\frac{2}{3} < x < 0$
- E $-\frac{2}{3} < x < \frac{2}{3}, x \neq 0$
- $$\frac{3x-2}{\sqrt[3]{x^2}} = 3x^{1/3} - 2x^{-2/3}$$

$$x^{-2/3} + \frac{4}{3}x^{-5/3} < 0$$

$$x^{4/3} + \frac{4}{3}x^{1/3} < 0$$

$$x^{1/3}(x + \frac{4}{3}) < 0$$
 If $x > 0$ $x < -4/3$ \times contradiction
 If $x < 0$ $x > -4/3$ $-\frac{4}{3} < x < 0$

17. The volume V , of a soap bubble is modelled by the formula $V = (p - qt)^2$ $t \leq 0$

where p and q are positive constants and t is the time in seconds after a certain instant.

When $t = 1$, the volume of a soap bubble is 9cm^3 and at that instant its volume is decreasing at the rate of 6cm^3 per second. What is the value of $p + q$?

- A 2 B 3 **C** 5 D 6 E 9

$$V = (p - qt)^2 = p^2 - 2pqt + q^2t^2$$

$$9 = (p - q)^2 \quad p - q = \pm 3$$

$$V' = -2pq + 2q^2t$$

$$-2pq + 2q^2 = -6$$

$$q^2 - pq = -3$$

$$q(q - p) = -3$$

$$q - p = 3 \quad q = -1 \quad p = -4 \quad \times$$

$$q - p = -3 \quad q = 1 \quad p = 4 \quad \checkmark$$

18. The least possible value of the gradient of the curve $y = (x + a)^2(3x - a)$ at the point where $x = \frac{1}{2}$, as a varies is

- A -9 B $-\frac{25}{4}$ **C** -4 D $-\frac{5}{2}$ E $\frac{5}{4}$

$$= (x^2 + 2ax + a^2)(3x - a)$$

$$= 3x^3 + 6ax^2 + a^2 3x - ax^2 - 2a^2x - a^3$$

$$= 3x^3 + 5ax^2 + a^2x - a^3$$

$$y' = 9x^2 + 10ax + a^2$$

$$x = \frac{1}{2} \quad y' = \frac{9}{4} + 5a + a^2$$

$$= (a + \frac{5}{2})^2 - \frac{25}{4} + \frac{9}{4}$$

$$= (a + \frac{5}{2})^2 - 4$$

19. Consider the function $f(x) = x^3 - 3x^2 - 144x$

Which of the following statements are true?

I The gradient of the function is negative for $x < 0$.

II There is a local maximum at $x = 8$.

III There is a point of inflexion at $x = 1$.

A none of them

B I only

C II only

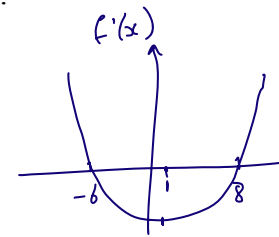
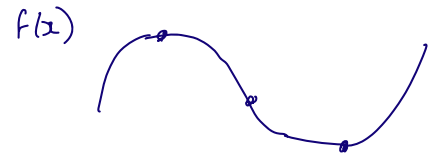
D III only

E I and II only

F I and III only

G II and III only

H I, II and III



$f'(x) > 0$ for $x < -6$
I is false

$$f'(x) = 3x^2 - 6x - 144$$

$$f''(x) = 6x - 6$$

$$f'(x) = 3(x^2 - 2x - 48)$$

$$= 3(x-8)(x+6)$$

Stat points $x = 8$ $x = -6$

$f''(8) = 48 - 6 > 0$ so minimum II is false

$f''(1) = 0$ - could be point of inflexion (even though not a stat. point)

$f''(0) < 0$ concave
 $f''(2) > 0$ convex
 change of sign so III is true

20. A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$$3x^4 - 4x^3 - 12x^2 + 20 = k \quad \text{has four distinct real roots}$$

A no values of k

B $-12 < k < 15$

C $15 < k < 20$

D $k > 15$

E $7 < k < 20$

$$y = 3x^4 - 4x^3 - 12x^2 + 20$$

T.P.s $(0, 20)$

$$y' = 12x^3 - 12x^2 + 24x = 0$$

$(-1, 15)$

$$x(x^2 - x + 2) = 0$$

$(2, -12)$

$$x(x-2)(x+1) = 0$$

$$3 + 4 - 12 + 20$$

$y = k$ 4 solutions

$$48 - 32 - 48 + 20$$

$15 < k < 20$

