1. Given that p and q are non-zero integers, the expression $\dfrac{(36^{p-q})(3^q)}{(12^{2p-q})(6^p)}$

   is an integer if:

   A $\quad p < 0$
   B $\quad q < 0$
   C $\quad p > 0$ and $q < 0$
   D $\quad p > 0$ and $q > 0$
   E $\quad p > 0$
   F $\quad q > 0$

$$= \frac{2^{2p-2q} \cdot 3^{2p-2q} \cdot 3^{q}}{2^{4p-2q} \cdot 3^{2p-q} \cdot 2^{p} \cdot 3^{p}}$$

$$= 2^{2p-2q-4p+2q-p} \cdot 3^{2p-2q+q-2p+q-p}$$

$$= 2^{-3p} \cdot 3^{-p}$$ integer for $p < 0$ all q

2. Given that $m = 7^8$ and $n = 8^7$ which expression represents 56^{56}

   A $\quad mn$
   B $\quad (mn)^{56}$
   C $\quad m^7 n^8$
   D $\quad 8m^7 + 7n^8$
   E $\quad (8m)^7(7n)^8$

$7^8 = m \quad 8^7 = n$

$7^{56} = m^7 \quad 8^{56} = n^8$

$56^{56} = 7^{56} \times 8^{56}$

$= \underline{m^7 n^8}$

3. Find the set of values of x that satisfy both the following inequalities:

$$\frac{4x+1}{x-1} < 3 \qquad (x+2)(x-4) > 0$$

   A $\quad x < -4$
   B $\quad x > -4$
   C $\quad -2 < x < 1$
   D $\quad -4 < x < 4$
   E $\quad -4 < x < -2$

$\frac{4x+1}{x-1} - 3 < 0$

$\frac{4x+1-3x+3}{x-1} < 0$

$\frac{x+4}{x-1} < 0$

either $x+4 > 0$, $x-1 < 0$

$\underline{-4 < x < 1}$

or $x+4 < 0$, $x-1 > 0$

no solutions

$(x+2)(x-4) > 0$

$x^2 - 2x > 8$

$\underline{x < -2 \quad x > 4}$

$-4 \quad -2 \quad 1 \quad 4$

together

$-4 < x < -2$

4. Find the set of values of x that satisfy the following inequality:

$$\frac{3}{x+3} > \frac{x-4}{x}$$

A $\quad -3 < x < 6$

B $\quad -2 < x < 6$

C $\quad -3 < x < -2 \quad$ and $\quad 0 < x < 6$

D $\quad 0 < x < 2 \quad$ and $\quad 3 < x < 6$

E $\quad 2 < x < 3 \quad$ and $\quad 5 < x < 6$

-ve quartic

$3(x+3)x^2 > (x-4)(x+3)^2 x$

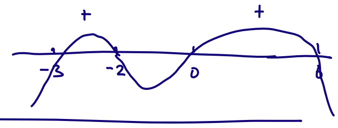
$(x+3)x\,(3x - (x^2-x-12)) > 0$

$(x+3)x\,(12+4x-x^2) > 0$

$(x+3)x(6-x)(x+2) > 0$

$-3 < x < -2$

$0 < x < 6$



or $\frac{3}{x+3} - \frac{x-4}{x} > 0$

$\frac{3x - x^2 + x + 12}{x(x+3)} > 0$

$\frac{x^2-4x-12}{x(x+3)} < 0$

$\frac{(x-6)(x+2)}{x(x+3)} < 0$

$-3 \quad -2 \quad 0 \quad 6$

$+ \quad - \quad + \quad - \quad +$

5. Find the set of values of x that satisfy the following inequality, where p is a positive constant:

$$\frac{x+p}{x+4p} < \frac{p}{x}$$

A $\quad -2p < x < 2p$

B $\quad 0 < x < 2p$

C $\quad x < -4p\,,\ x > 0$

D $\quad -4p < x < -2p\,,\ 0 < x < 2p$

E $\quad -4p < x < 0\,,\ x > 2p$

$\frac{x+p}{x+4p} - \frac{p}{x} < 0$

$\frac{x^2+px-px-4p^2}{x(x+4p)} < 0$

$\frac{(x-2p)(x+2p)}{x(x+4p)} < 0$

$-4p \quad -2p \quad 0 \quad 2p$

$+ \quad - \quad + \quad - \quad +$

$-4p < x < -2p$

$0 < x < 2p$

6. A cubic curve has equation $y = x^3 + kx - 2$ where k is a constant.
What value of k gives this curve exactly two distinct real roots

A $\quad -3$

B $\quad -2$

C $\quad -1$

D $\quad 1$

E $\quad 3$

2 distinct roots $\Rightarrow$ repeated root (A) and single root (B)

$(x-A)^2(x-B) = (x^2 - 2Ax + A^2)(x-B)$

$= x^3 - x^2(2A+B) + x(A^2+2AB) - A^2B$

Comparing coefficients:

$2A+B=0$

$B=-2A$ (1)

$A^2+2AB=k$ (2)

$A^2B=2$ (3)

(1) in (3) $-2A^3=2$

$A^3=-1$

$A=-1 \quad B=2$

$(x+1)^2(x-2)$

(2) $\quad k = 1+2(-1)(2) = -3$

7. The equation $2x^2 + 9x - k = 0$ where k is a constant has two distinct real roots.

One root is 4 more than the other root. The value of k is

A $\frac{55}{8}$ B $\frac{9}{2}$ C $-\frac{17}{8}$ D $-\frac{17}{4}$ E $-\frac{55}{8}$

Let roots $= a,\ a+4$

$(x-a)(x-a-4)=0$

$x^2 - (a+a+4)x + a(a+4) = 0$

$2x^2 - (4a+8)x + 2a(a+4) = 0$

$4a+8 = -9$

$a = -\frac{17}{4}$

$-k = -\frac{17}{2}\left(-\frac{1}{4}\right)$

$k = -\frac{17}{8}$

8. Find the minimum value of $2(2^{sinx}) - 4^{sinx} + \frac{10}{3}$

A $\frac{10}{3}$ B $\frac{13}{3}$ C $\frac{49}{12}$ D $\frac{20}{3}$ E 0

Let $y = 2^{\sin x}$

$-1 \le \sin x \le 1$

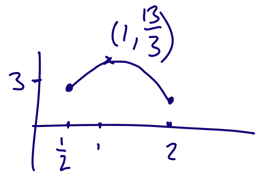
$\frac{1}{2} \le y \le 2$

$2y - y^2 + \frac{10}{3}$

$\frac{10}{3} - (y^2 - 2y)$

$\frac{10}{3} - [(y-1)^2 - 1]$

$\frac{13}{3} - (y-1)^2$



Min at $y = 2$

$\frac{13}{3} - 1 = \frac{10}{3}$

9. When $(2x^2 + 6x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$ the remainder is 28.

The value of p is

A 3 B 2 C $\frac{7}{4}$ D $\frac{3}{2}$ E $\frac{28}{5}$

Let $f(x) = (2x^2 + 6x - 3)(px - 1)$

$f(-1) = 28$

$(2 - 6 - 3)(-p - 1) = 28$

$7(p+1) = 28$

$p = 3$

10. The simultaneous equations below have two distinct real solutions

$$3x^2 - xy = 4 \text{ and } \quad 2x - y = p \quad \text{where } p \text{ is a real constant}$$

What are the values that p can take

$y = 2x - p$

A there are no possible values for p

B $p < -4, p > 4$

C $-4 < p < 4$

D p can take any value

$3x^2 - 2x^2 + xp = 4$

$x^2 + px - 4 = 0$

$\Delta > 0$

$p^2 + 4(4) > 0$

$p^2 + 16 > 0$

true for all p

11. What is the sum of the solutions of the following equation

$$|x| - 3 = |2x + 12|$$

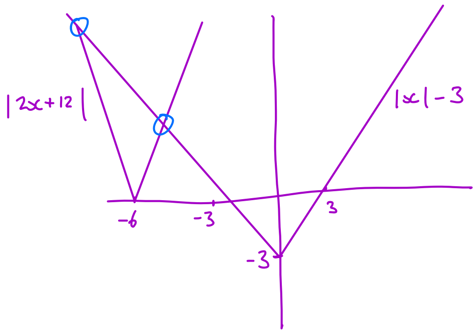
A - 14

B - 4

C 0

D 4

E 14



$-x - 3 = 2x + 12$

$3x = -15$

$x = -5$

$-x - 3 = -2x - 12$

$x = -9$

$-5 - 9 = -14$

12. How many solutions are there to the following equation:

$$|x| + |x - 1| = |x^3|$$

A 0

B 1

C 2

D 3

E 4

$|x-1| = |x^3| - |x|$

$x > 0 \quad |x^3| - |x| = x^3 - x$

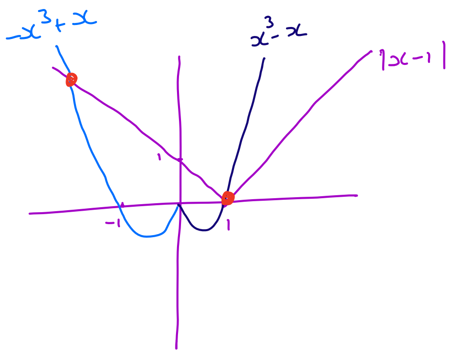
$= x(x^2 - 1)$

$= x(x-1)(x+1)$

$x < 0 \quad |x^3| - |x| = -x^3 + x$

$= -x(x^2 - 1)$

$= -x(x-1)(x+1)$



13. Given that $\left(a^3+\frac{3}{b^3}\right)\left(b^3-\frac{3}{a^3}\right)=2\sqrt{3}$ where a, b are real numbers,

then the least value of ab is

A $-\sqrt{3}$

B $\sqrt{3}$

C $-3\sqrt{3}$

D $3\sqrt{3}$

E $-3^{\frac{1}{6}}$

F $3^{\frac{1}{6}}$

$a^3b^3+3-3-\frac{9}{a^3b^3}=2\sqrt{3}$

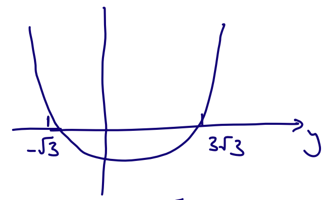
$x=ab \qquad x^3-\frac{9}{x^3}=2\sqrt{3}$

$y=x^3$

$y=(ab)^3 \qquad y-\frac{9}{y}=2\sqrt{3}$

$y^2-2\sqrt{3}y-9=0$

$(y-3\sqrt{3})(y+\sqrt{3})=0$



Least $y=-\sqrt{3}$

$x=-3^{1/6}$

min value $ab=-3^{1/6}$

14. The function f is defined such that $3f(x)+2f(-x)=5x-10$
find the value of $f(1)$

A 0

B 1

C 2

D 3

E 4

$3f(x)+2f(-x)=5x-10$

$3f(-x)+2f(x)=-5x-10$

$9f(x)+6f(-x)=15x-30$

$4f(x)+6f(-x)=-10x-20$

$5f(x)=25x-10$

$f(x)=5x-2$

$f(1)=3$

15. The function f satisfies $2f(x)-f\left(\frac{2x+3}{x-2}\right)=2x-2, \quad x\in\mathbb{R}$

What is the value of $f(9)$

A 16

B 12

C 8

D -12

E -16

$x=9 \qquad 2f(9)-f(3)=16 \quad ①$

$x=3 \qquad 2f(3)-f(9)=4$

$① \times 2 \qquad 4f(9)-2f(3)=32$

$2f(3)-f(9)=4 \quad +$

$3f(9)=36$

$f(9)=12$

$f(3)=8$