1. Solve the following trigonometric equations in the range given

a) $\cos(2\theta + 30) = 0.5 \quad 0 \le \theta \le 360°$ $\quad 30 \le 2\theta + 30 \le 750$

$2\theta + 30 = 60, 300, 420, 660$
$2\theta = 30 \quad 270 \quad 390 \quad 630$
$\theta = 15, 135, 195, 315$

b) $\tan(5\theta - 35) = \sqrt{3} \quad 0 \le \theta \le 90°$ $\quad -35 \le 5\theta - 35 \le 415$

$5\theta - 35 = 60 \quad 240$
$5\theta = 95 \quad 275$
$\theta = 19 \quad 55$

c) $2\sin^2\theta + 3\cos\theta = 0 \quad 0 \le \theta \le 360°$

$2(1 - \cos^2\theta) + 3\cos\theta = 0$
$2 - 2\cos^2\theta + 3\cos\theta = 0$
$2\cos^2\theta - 3\cos\theta - 2 = 0$
$(2\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = 2$ no solutions $\quad \cos\theta = -\frac{1}{2}$ $\theta = 120, 240$

d) $\sin(3\theta + 72) = \cos 48 \quad 0 \le \theta \le 180°$ $\quad 72 \le 3\theta + 72 \le 612$

$\cos A = \sin(90 - A)$
$\cos 48 = \sin(90 - 48)$

$\sin(3\theta + 72) = \sin(90 - 48)$
$3\theta + 72 = (42), 138, 402, 498$
$3\theta = \quad 66 \quad 330 \quad 426$
$\theta = \quad 22 \quad 110 \quad 142$

e) $\dfrac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta \quad 0 \le \theta \le 360°$

$3 + \sin^2\theta = 3\cos^2\theta - 6\cos\theta$
$3 + 1 - \cos^2\theta = 3\cos^2\theta - 6\cos\theta$
$4\cos^2\theta - 6\cos\theta - 4 = 0$

$2\cos^2\theta - 3\cos\theta - 2 = 0$
$(2\cos\theta + 1)(\cos\theta - 2) = 0$
$\cos\theta = -\frac{1}{2} \quad \cos\theta = 2$
$\theta = 120, 240$ no solutions

f) $\cos^2(2x) + \sqrt{3}\sin(2x) - \dfrac{7}{4} = 0 \quad 0 \le x \le 360°$

$1 - \sin^2 2x + \sqrt{3}\sin 2x - \frac{7}{4} = 0$
$4\sin^2 2x - 4\sqrt{3}\sin 2x + 3 = 0$
$(2\sin 2x - \sqrt{3})(2\sin 2x - \sqrt{3}) = 0$

$\sin 2x = \frac{\sqrt{3}}{2}$
$2x = 60, 120, 420, 480$
$x = 30, 60, 210, 240$

OR $\sin 2x = \dfrac{4\sqrt{3} \pm \sqrt{48 - 48}}{8} = \dfrac{\sqrt{3}}{2}$

if you don't spot the factorisation

2. Solve, in radians, the following trigonometric equations, giving your answers in terms of π.

a) $\cos 2x = \cos\frac{2\pi}{5} \quad 0 \le x \le 2\pi$

$0 \le 2x \le 4\pi$

$2x = \frac{2\pi}{5}, \frac{8\pi}{5}, \frac{12\pi}{5}, \frac{18\pi}{5}$

$x = \frac{\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{9\pi}{5}$

b) $8\sin(\frac{\pi}{3} - 2x) = 4 \quad 0 \le x \le 2\pi$

$-\frac{\pi}{3} \le 2x - \frac{\pi}{3} \le \frac{11\pi}{3}$

$\sin\left(\frac{\pi}{3} - 2x\right) = \frac{1}{2}$

$\sin\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$ (as sine odd)

$2x - \frac{\pi}{3} = -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}$

$2x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{7\pi}{2}$

$x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{4}$

c) $\sin^2\frac{3\theta}{2} = \frac{1}{2} \quad 0 \le \theta \le 2\pi$

$0 \le \frac{3\theta}{2} \le 3\pi$

$\sin\frac{3\theta}{2} = \pm\sqrt{\frac{1}{2}}$

$\frac{3\theta}{2} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{6}$

$\frac{3\theta}{2} = \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

d) $2\sin^2 y - 5\cos y + 1 = 0 \quad 0 \le y \le 2\pi$

$2 - 2\cos^2 y - 5\cos y + 1 = 0$

$2\cos^2 y + 5\cos y - 3 = 0$

$(2\cos y - 1)(\cos y + 3) = 0$

$\cos y = \frac{1}{2}$

$y = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos y = -3$

no solutions

e) $\tan^4 x - \tan^2 x = 6 \quad 0 \le x \le 2\pi$

$\tan^4 x - \tan^2 x - 6 = 0$

$(\tan^2 x - 3)(\tan^2 x + 2) = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

$\tan^2 x + 2 = 0$

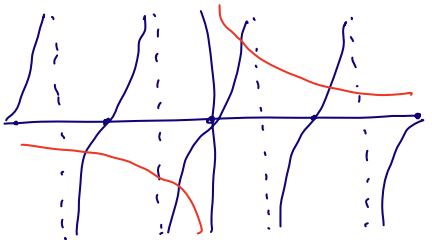
no solutions

3. Find the number of solutions to the following equations:

a) $x \tan x = 2$ $\quad -2\pi \le x \le 2\pi$

$\tan x = \frac{2}{x}$

sketch $y = \tan x$ $y = \frac{2}{x}$ + count intersections

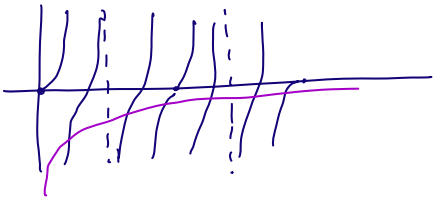


4 solutions

b) $x \sin 3x + \cos 3x = 0$ $\quad 0 \le x \le 2\pi$

$x \sin 3x = -\cos 3x$

$\tan 3x = -\frac{1}{x}$



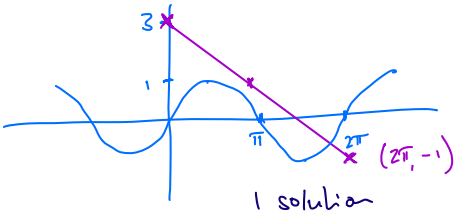
6 solutions

c) $2x + \pi \sin x = 3\pi$

$\sin x = -\frac{2}{\pi}x + 3$

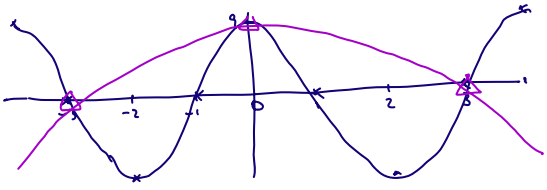
straight line

passing through $(0,3)$ $(\pi,1)$ $(2\pi,-1)$



1 solution

d) $(x+3)(3-x) = 9\cos\frac{\pi}{2}x$



3 solutions

e) $2^x + 2\sin(\pi x) = 3$

$\sin(\pi x) = \frac{3 - 2^x}{2}$

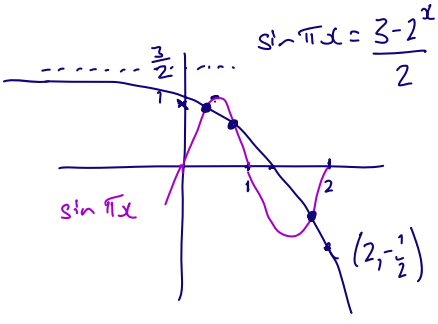
$y = 2^x$ $(0,1)$

reflect in x-axis $y = -2^x$ $(0,-1)$

stretch in y direction SF $\frac{1}{2}$ $y = \frac{-2^x}{2}$ $(0,-\frac{1}{2})$

translate in y direction by $+\frac{3}{2}$ $y = \frac{3-2^x}{2}$ $(0,1)$

$y = 0$ $2^x = 3$ $1 < x < 2$



3 solutions

4. Find the complete set of values for which:

a) $y \geq 0$ where $y = \tan x \cos 2x$ $\quad 0 \leq x \leq \pi$ $\quad x \neq \frac{\pi}{2}$

	$0 - \frac{\pi}{4}$	$\frac{\pi}{4} - \frac{\pi}{2}$	$\frac{\pi}{2} - \frac{3\pi}{4}$	$\frac{3\pi}{4} - \pi$
$\tan x$	+	+	−	−
$\cos 2x$	+	−	−	+

$0 \leq x \leq \frac{\pi}{4}, \quad \frac{\pi}{2} < x \leq \frac{3\pi}{4}$

b) $y \geq 0$ where $y = (1 - 2\sin x)\cos x$ $\quad 0 \leq x \leq \pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \quad \frac{5\pi}{6}$

	$0 - \frac{\pi}{6}$	$\frac{\pi}{6} - \frac{\pi}{3}$	$\frac{\pi}{3} - \frac{\pi}{2}$	$\frac{\pi}{2} - \frac{2\pi}{3}$	$\frac{2\pi}{3} - \frac{5\pi}{6}$	$\frac{5\pi}{6} - \pi$
$1 - 2\sin x$	+	−	−	−	−	+
$\cos x$	+	+	+	−	−	−

$0 \leq x \leq \frac{\pi}{6}$

$\frac{\pi}{2} \leq x \leq \frac{5\pi}{6}$

c) $y \leq 0$ where $y = (1 + \cos 2x)\cos 2x$ $\quad 0 \leq x \leq \pi$

$\cos 2x = -1$

$2x = \pi$

$x = \frac{\pi}{2}$

	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$1 + \cos 2x$	+	+	+	+	
$\cos 2x$	+	−	−	+	

$\cos 2x = 0$

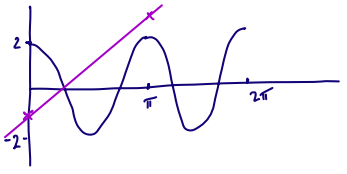
$2x = \frac{\pi}{2} \quad \frac{3\pi}{2}$

$\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

d) $2\cos 2x \leq \frac{4}{\pi}x - 1$ $\quad 0 \leq x \leq 2\pi$

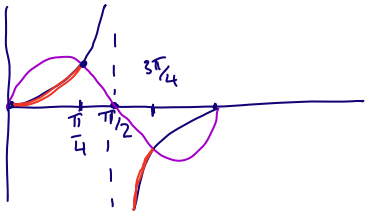
$y = \frac{4}{\pi}x - 1$ straight line passing through $(0, -1)$ $(\pi, 3)$

$y = 0 \quad (\frac{\pi}{4}, 0)$



$\frac{\pi}{4} \leq x \leq 2\pi$

e) $\tan x \leq \sin 2x$ $\quad 0 \leq x \leq \pi$



$x = \frac{\pi}{4} \quad \tan x = 1, \quad \sin 2x = 1$

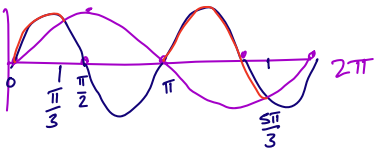
$x = \frac{3\pi}{4} \quad \tan x = -1, \quad \sin 2x = -1$

$0 \leq x \leq \frac{\pi}{4}$

$\frac{\pi}{2} < x \leq \frac{3\pi}{4}$

f) $\sin 2x > \sin x$ $\quad 0 \leq x \leq 2\pi$

Find the fraction of the interval $0 \leq x \leq 2\pi$ for which this inequality is true.



$\sin 2x = \sin x$ at $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$0 < x < \frac{\pi}{3}, \quad \pi < x < \frac{5\pi}{3}$

$\text{fraction} = \frac{\frac{1}{3}\pi + \frac{2}{3}\pi}{2\pi} = \frac{1}{2}$

5. Find the equation of the new curves after the following transformations:

a) The curve $y = \cos x$ is stretched in the horizontal direction by a scale factor of $\frac{1}{2}$, and the resulting curve is translated by 4 units in the positive y-direction.

$f(x) \to f(2x) \to f(2x) + 4$ $\quad y = \cos 2x + 4$

b) The curve $y = \sin x$ is stretched in the vertical direction by a scale factor of 2, followed by a translation by $\frac{\pi}{3}$ units in the positive x-direction.

$f(x) \to 2f(x) \to 2f\left(x - \frac{\pi}{3}\right)$ $\quad y = 2\sin\left(x - \frac{\pi}{3}\right)$

c) The curve $y = \tan x$ is stretched in the horizontal direction by a scale factor of 2, followed by a reflection in the x-axis, followed by a translation by 3 units in the positive y-direction.

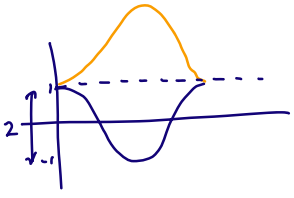
$f(x) \to f\left(\frac{1}{2}x\right) \to -f\left(\frac{1}{2}x\right) \to 3 - f\left(\frac{1}{2}x\right)$ $\quad y = 3 - \tan\left(\frac{1}{2}x\right)$

d) The curve $y = \sin x$ is reflected in the y-axis, translated by 2 units in the negative y-direction and then translated by $\frac{\pi}{4}$ units in the positive x-direction.

$f(x) \to f(-x) \to f(-x) - 2 \to f\left(-\left(x - \frac{\pi}{4}\right)\right) - 2$

$y = \sin\left[-\left(x - \frac{\pi}{4}\right)\right] - 2$ or $y = -\sin\left(x - \frac{\pi}{4}\right) - 2$

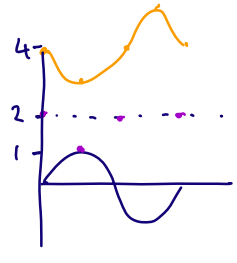
e) The curve $y = \cos x$ is reflected in the line $y = 1$



Reflection in $y = k$ $\quad f(x) \to 2k - f(x)$

$f(x) \to 2 - f(x)$ $\quad y = 2 - \cos x$

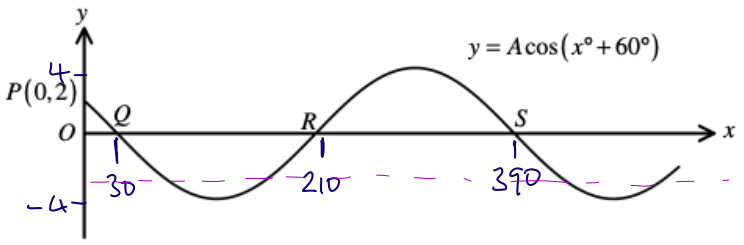
f) The curve $y = \sin x$ is reflected in the line $y = 2$ followed by a translation by $\frac{\pi}{3}$ units in the negative x-direction.



$f(x) \to 4 - f(x) \to 4 - f\left(x + \frac{\pi}{3}\right)$

$y = 4 - \sin\left(x + \frac{\pi}{3}\right)$

6a) The figure shows part of the graph of the curve with equation $y = A\cos(x + 60)$



$2 = A\cos 60 \quad A = 4$

$y = 4\cos(x+60)$

The point P (0,2) lies on the curve. Find the value of A and the coordinates of Q, R, and S.

Find the coordinates of the points where the straight line with equation $y = -2\sqrt{3}$ meets this graph.

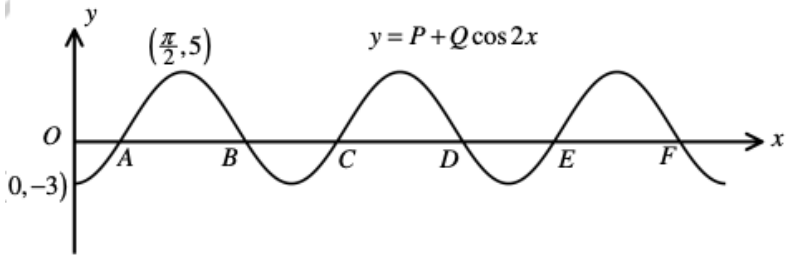
$4\cos(x+60) = -2\sqrt{3}$

$\cos(x+60) = -\frac{\sqrt{3}}{2}$

$x + 60 = 150 \quad 210 \quad 510 \quad 570$

$x = 90 \quad 150 \quad 450 \quad 510$

6b) The figure shows part of the graph of the curve with equation $y = P + Q\cos 2x \qquad x \geq 0$



The points (0 , -3) and $(\frac{\pi}{2}, 5)$ lie on the curve. Find the value of P and Q.

Find the coordinates of the points where the straight line with equation $y = 3$ meets this graph for $0 \leq x \leq 2\pi$.

$-3 = P + Q$

$5 = P - Q$

$P = 1$

$Q = -4$

$3 = 1 - 4\cos 2x$

$\cos 2x = -\frac{1}{2}$

$0 \leq 2x \leq 4\pi$

$2x = \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad \frac{8\pi}{3} \quad \frac{10\pi}{3}$

$x = \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad \frac{5\pi}{3}$

$y = 1 - 4\cos 2x$

6c) A curve has equation $y = A + k \sin x$, $0 \le x \le 2\pi$, where A and k are non-zero constants.

Given that the curve passes through $(\frac{\pi}{6}, 1)$ and $(\frac{7\pi}{6}, 5)$, find the minimum and maximum value of y.

$1 = A + \frac{1}{2}k$

$5 = A - \frac{1}{2}k$

$A = 3 \quad k = -4$

$y = 3 - 4\sin x$

$\sin x = -1 \quad y = 7$

$\sin x = 1 \quad y = -1$



$y = 4\sin x$



$y = -4\sin x$



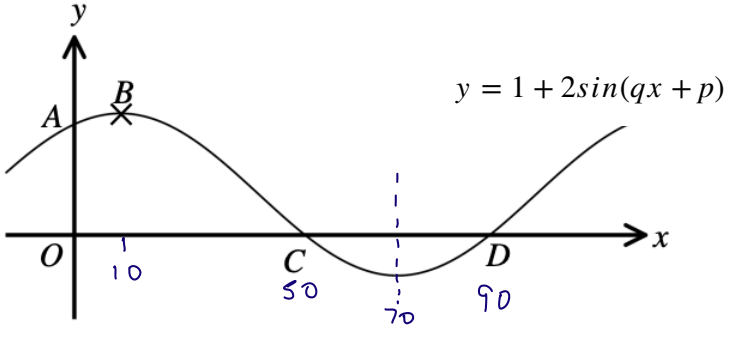
$y = 3 - 4\sin x$

6d) The figure shows part of the graph of $y = 1 + 2\sin(qx + p)$,

where q and p are positive constants, with $0 < p < 90$, $0 < q < 5$

The graph crosses the y-axis at the point A $(0 , 1 + \sqrt{3})$ and the x-axis at the points C (50 , 0) and D, and point B is a maximum point on the curve.

Find the values of p and q, and the coordinates of B and D.



$y = 1 + 2\sin(3x + 60)$

$= 1 + 2\sin 3(x + 20)$

$1 + \sqrt{3} = 1 + 2\sin p$

$\sin p = \frac{\sqrt{3}}{2}$

$p = 60$

$0 = 1 + 2\sin(50q + 60)$

$\sin(50q + 60) = -\frac{1}{2}$

$50q + 60 = 210$

$50q = 150$

$q = 3$

B $(10 , 3)$ D $(90, 0)$

7.

a) Find the greatest value of the function $f(x) = (2\cos^2(4x+9) - 5)^2$ for $x \in \mathbb{R}$

$0 \le \cos^2(4x+9) \le 1$

$0 \le 2\cos^2(4x+9) \le 2$

$-5 \le 2\cos^2(4x+9) \le -3$

max = 25

b) Find the largest value achieved by $f(x) = 3\cos^2 x + 2\sin x + 1$ for $x \in \mathbb{R}$

$3 - 3\sin^2 x + 2\sin x + 1$

$-3\sin^2 x + 2\sin x + 4$

$-3\left(\sin^2 x - \frac{2}{3}\sin x - \frac{4}{3}\right)$

$-3\left[\left(\sin x - \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{4}{3}\right]$

$-3\left[\left(\sin x - \frac{1}{3}\right)^2 - \frac{13}{9}\right]$

$-3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$

Max at $\sin x = \frac{1}{3}$ $= \frac{13}{3}$

c) Find the largest value of $f(x) = (4\sin^2 x + 4\cos x + 1)^2$ for $x \in \mathbb{R}$

$= (4 - 4\cos^2 x + 4\cos x + 1)^2$

$= (5 + 4\cos x - 4\cos^2 x)^2$

$= \left(-4\left[\cos^2 x - \cos x - \frac{5}{4}\right]\right)^2$

$= \left(-4\left[\left(\cos x - \frac{1}{2}\right)^2 - \frac{3}{2}\right]\right)^2$

$= \left(6 - 4\left(\cos x - \frac{1}{2}\right)^2\right)^2$

Max when $\cos x = 1/2$ 36

d) Find the minimum value of the function $f(x) = 9\cos^4 x - 12\cos^2 x + 7$ for $x \in \mathbb{R}$

$= 9\left[\cos^4 x - \frac{4}{3}\cos^2 x + \frac{7}{9}\right]$

$= 9\left[\left(\cos^2 x - \frac{2}{3}\right)^2 - \frac{4}{9} + \frac{7}{9}\right]$

$= 9\left(\cos^2 x - \frac{2}{3}\right)^2 + 3$

Min at $\cos^2 x = \frac{2}{3}$ 3

e) Find the maximum value of $9^{\sin x} - 4(3^{\sin x}) + \frac{13}{3}$ for $x \in \mathbb{R}$

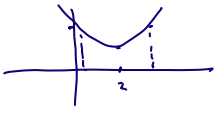
$y = 3^{\sin x}$

$-1 \le \sin x \le 1$

$\frac{1}{3} \le y \le 3$

$y^2 - 4y + \frac{13}{3}$

$(y-2)^2 + \frac{1}{3}$



$y = \frac{1}{3}$ $\frac{25}{9} + \frac{3}{9} = \frac{28}{9}$ ← Max

$y = 3$ $1 + \frac{1}{3} = \frac{4}{3}$

f) Find the minimum value of $f(x) = \dfrac{\cos x + 4}{9 + 6\cos x - \sin^2 x}$ for $x \in \mathbb{R}$

$f(x) = \dfrac{\cos x + 4}{9 + 6\cos x + \cos^2 x - 1}$

$= \dfrac{\cos x + 4}{\cos^2 x + 6\cos x + 8}$

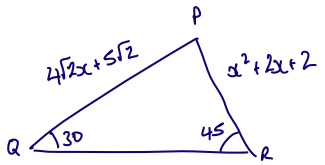
$= \dfrac{\cos x + 4}{(\cos x + 4)(\cos x + 2)}$

$= \dfrac{1}{\cos x + 2}$

Min when $\cos x = 1$

$\frac{1}{3}$

8a) In triangle PQR, PR has length $(x^2 + 2x + 2)$, PQ has length $(4\sqrt{2}x + 5\sqrt{2})$, angle PRQ is 45°, and angle PQR is 30°. Find the sum of all possible values of x.



use sine rule

$\frac{x^2+2x+2}{\sin 30} = \frac{(4x+5)\sqrt{2}}{\sin 45}$

$\frac{2(x^2+2x+2)}{1} = \frac{2(4x+5)\sqrt{2}}{\sqrt{2}}$

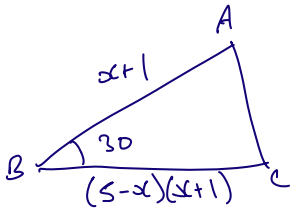
$x^2 + 2x + 2 = 4x + 5$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

Sum of possible values $= 2$

8b) A triangle ABC has $AB = x + 1$, $BC = (5 - x)(x + 1)$, and angle $ABC = 30°$. What is the greatest area of the triangle?



$-1 < x$
$-1 < x < 5$ $\Big]$ $-1 < x < 5$

$A = \frac{1}{2}(x+1)(x+1)(5-x)\left(\frac{1}{2}\right)$

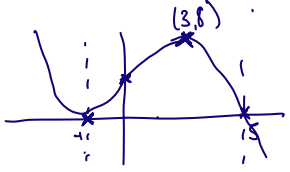
$A = \frac{1}{4}(x^2+2x+1)(5-x)$

$= \frac{1}{4}(5 + 9x + 3x^2 - x^3)$

$A' = \frac{1}{4}(9 + 6x - 3x^2) = 0$

$x^2 - 2x + 3 = 0$

$(x+1)(x-3) = 0$

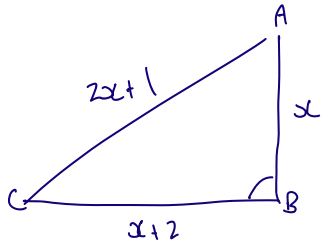


Max $x = 3$

$A = \frac{1}{2}(4)(8)\left(\frac{1}{2}\right) = 8$

8c) A triangle ABC has $AB = x$, $BC = x + 2$, and $AC = 2x + 1$. For which values of x is angle ABC obtuse?

when $AC^2 > AB^2 + BC^2$



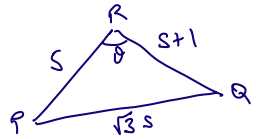
$(2x+1)^2 > x^2 + (x+2)^2$

$4x^2 + 4x + 1 > 2x^2 + 4x + 4$

$2x^2 - 3 > 0$

$x^2 > \frac{3}{2}$ $\quad x > \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$

8d) The lengths of the sides QR, RP and PQ in triangle PQR are $s + 1$, s, and $\sqrt{3}s$. Find the full range of values of s that make angle PRQ an obtuse angle.



$\cos\theta = \frac{s^2 + s^2 + 2s + 1 - 3s^2}{2s(s+1)} = \frac{-s^2 + 2s + 1}{2s(s+1)} = -\frac{s^2 - 2s - 1}{2s(s+1)}$

For obtuse angle need $-1 < \cos\theta < 0$ $\quad 0 < \frac{s^2 - 2s - 1}{2s(s+1)} < 1$

Need $s > 0$

$\therefore 2s(s+1) > 0$ (OK as $2s(s+1) > 0$)

So $s^2 - 2s - 1 > 0$ and $s^2 - 2s - 1 < 2s^2 + 2s$

$s = \frac{2 \pm \sqrt{4+4}}{2}$

$s = 1 \pm \sqrt{2}$

$s > 1 + \sqrt{2}$

$s^2 + 4s + 1 > 0$

$s = \frac{-4 \pm \sqrt{16-4}}{2}$

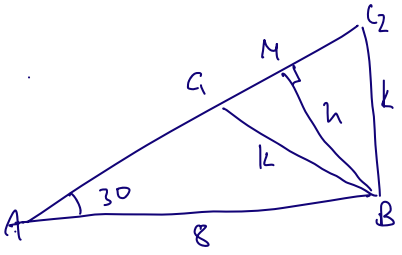
$s = -2 \pm \sqrt{3}$

$s > -2 + \sqrt{3}$

$\therefore s > 1 + \sqrt{2}$

9a) In triangle ABC, $AB = 8$, $BC = k$, angle $BAC = 30°$.

For which values of k are there exactly two non-congruent triangles satisfying these conditions.



AMB: $\sin 30 = \frac{h}{8}$

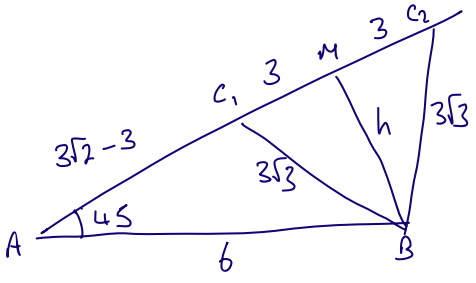
$h = 4$

For 2 triangles need $h < k < 8$

$4 < k < 8$

9b) In triangle ABC, $AB = 6$, $BC = 3\sqrt{3}$, angle $BAC = 45°$.

Two non-congruent triangles satisfy these conditions. What is the ratio of their areas?



$\sin 45 = \frac{h}{6}$

$h = 3\sqrt{2}$

$AM = 3\sqrt{2}$

$27 = h^2 + C_1M^2$

$27 = 18 + C_1M^2$

$C_1M = 3$

$AC_1 = 3\sqrt{2} - 3$

$AC_2 = 3\sqrt{2} + 3$

Ratio of areas = Ratio of sides

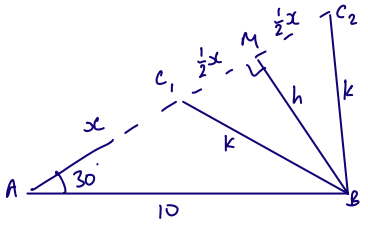
$3\sqrt{2} - 3 : 3\sqrt{2} + 3$

Area $ABC_1 = \frac{1}{2} \times 6 \times AC_1 \times \sin 45$

Area $ABC_2 = \frac{1}{2} \times 6 \times AC_2 \times \sin 45$

9c) A triangle ABC has $AB = 10$, angle $BAC = 30°$ and $BC = k$ cm, where k is a constant.

Of the two possible triangles that could be drawn, the larger triangle has side AC twice as long as side AC in the smaller triangle. Find the value of k.



AMB: $\sin 30 = \frac{h}{10}$ $h = 5$

$AM^2 = 10^2 - 5^2 = 75$

$\frac{3}{2}x = 5\sqrt{3}$

$x = \frac{10\sqrt{3}}{3}$ $x^2 = \frac{100}{3}$

BMC $k^2 = \frac{x^2}{4} + 25$

$= \frac{25}{3} + \frac{75}{3}$

$k = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

OR using Cosine Rule

$k^2 = 10^2 + x^2 - 20x\left(\frac{\sqrt{3}}{2}\right)$

$= 100 + x^2 - 10\sqrt{3}x$

$k^2 = 10^2 + 4x^2 - 40x\left(\frac{\sqrt{3}}{2}\right)$

$= 100 + 4x^2 - 20\sqrt{3}x$

$x^2 - 10\sqrt{3}x = 4x^2 - 20\sqrt{3}x$

$10\sqrt{3}x = 3x^2$

$x = 0$

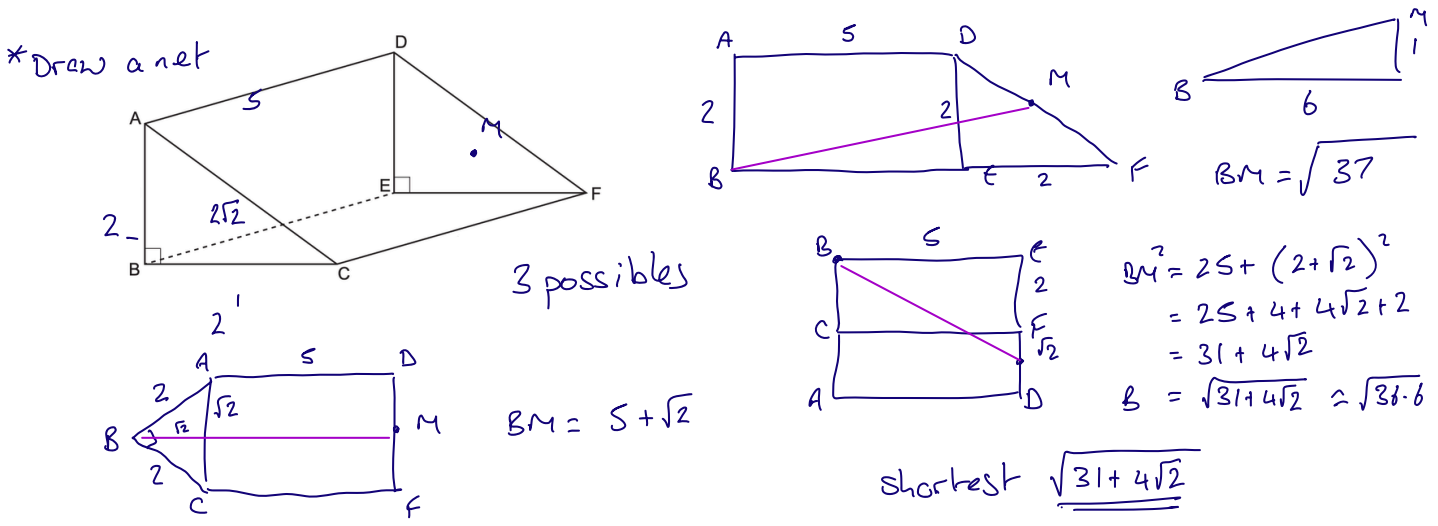
$x = \frac{10\sqrt{3}}{3}$

$k^2 = 100 + \frac{300}{9} - \frac{300}{3} = \frac{100}{3}$

$k = \frac{10\sqrt{3}}{3}$

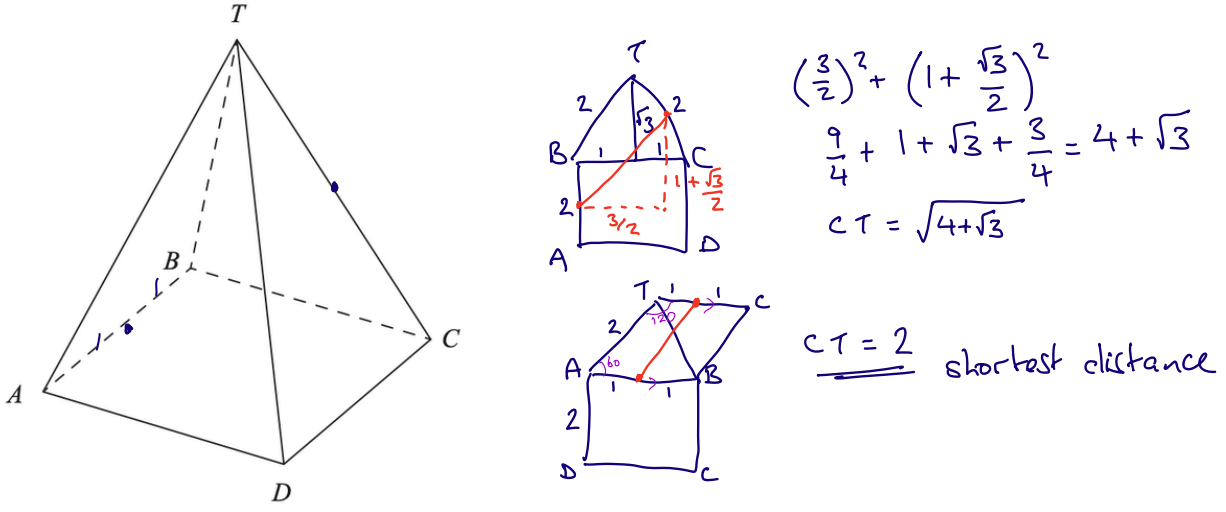
10a) A solid right angled triangular prism, with base $BEFC$, has $AB = BC = 2$m, and $AD = 5$m.

Find the shortest distance, in metres, along the outside of the prism from B to the midpoint of DF.



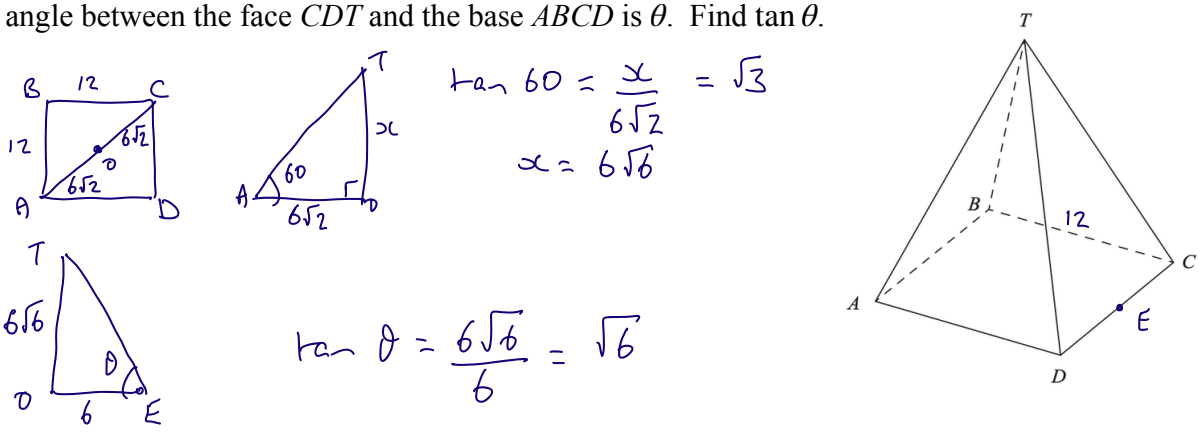
10b) A square based pyramid, with base $ABCD$, and vertex T has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the pyramid from the midpoint of AB to the midpoint of CT.



10c) A square based pyramid has base $ABCD$, where all the sides of the square are 12cm in length. The diagonals of the square intersect at O, and the vertex of the pyramid is at T, directly above O. Each of the sloping edges of the pyramid makes an angle of 60° with the base.

The angle between the face CDT and the base $ABCD$ is θ. Find $\tan\theta$.



10d) A triangular-based pyramid $PQRS$ has horizontal base PQR where

$$PQ = PR = 8\text{m} \quad \text{and} \quad RQ = 12\text{m}.$$

The vertex of the pyramid S lies directly above the level of PQR so that

$$SQ = SR = 10\text{m} \quad \text{and} \quad SP = 8\text{m}.$$

Find the shortest distance of S from the base PQR.

