

- 1) If $f(x)$ satisfies $f(x) = f(x^2)$ for all real x , which of the following must be true:
- A $f(4) = f(2)f(2)$
 - B $f(16) - f(-2) = 0$
 - C $f(0) = 0$
 - D $f(-2) + f(4) = 0$
- 2) If $a < 0$ and $b < c$, which of the following must be true?
- A $ab < c$
 - B $ac > b$
 - C $ab > 0$
 - D $ac < 0$
 - E $ab > ac$
- 3) Given that p and q are integers, and that three of the following statements are true, which is the false statement?
- A pq is even
 - B $p + q$ is even
 - C $2p + q^2$ is odd
 - D $p^2 + 2q$ is odd

4) Consider the following conjecture:

“If N is a positive integer that consists of the digit 2 followed by an even number of 1 digits, then N is a prime number”

Here are three numbers:

I	$N = 21$ (which equals 3×7)
II	$N = 211$ (a prime number)
III	$N = 21111$ (which equals 227×93)

Which of these provide(s) a counterexample to the original conjecture?

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

5) Which of the following is a necessary and sufficient condition for

$$\sum_{k=1}^n \tan\left(\frac{k\pi}{3}\right) = 0$$

- A $n = 3$
- B n is a multiple of 3
- C n is a multiple of 6
- D n is a multiple of 3 or n is 1 less than a multiple of 3
- E n is a multiple of 6 or n is 1 less than a multiple of 6

6) The real numbers a, b, c, d satisfy

$$0 < a + b < c + d \quad \text{and} \quad 0 < a + c < b + d$$

Which of the following must be true:

- I $a < d$
- II $b < c$
- III $a + b + c + d > 0$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

7) The arithmetic mean of five consecutive integers is an odd integer.

Which of the following must be true?

I The largest of the integers is even

II The sum of the integers is odd

III The difference between the largest and smallest of the integers is even.

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

8) Consider the following statement:

If $f(f(x)) = x$ **then** $f(x) = x$

Which function provides a counterexample:

A $f(x) = 1$

B $f(x) = x$

C $f(x) = \frac{1}{x}$

D $f(x) = x^2$

9) If a , b and c are integers, consider the statement $\frac{ab^2}{c}$ is a positive even integer (*)

Which of the following is a necessary but not sufficient condition for (*) to be true

I ab is even

II $ab > 0$

III c is even

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

10) For any real numbers a , b , and c where $a \geq b$, which of these three statements **must** be true?

I $-b \geq -a$

II $a^2 + b^2 \geq 2ab$

III $ac \geq bc$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

11) If x , a and b are positive integers such that when x is divided by a the remainder is b and when x is divided by b the remainder is $(a - 2)$, then which of the following must be true?

A a is even

B $b = a - 1$

C $x + b$ is divisible by a

D $a + 2 = b + 1$

12) Consider the following statement about positive integers a , b , c and d

If a divides $(b + c)$ and a divides $(c + d)$, then a divides $(b + d)$

Which of the following provide(s) a counterexample to the original conjecture?

I $a = 3, b = 4, c = 5, d = 8$

II $a = 5, b = 3, c = 2, d = 8$

III $a = 2, b = 3, c = 5, d = 7$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

13) If $\left| \frac{x}{4} \right| > 1$ which of the following must be true?

- I $x > 4$ II $x \neq 4$ III $x < -4$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

14) Consider the statement:.

(*) A whole number n is prime if it is 1 less or 5 less than a multiple of 8.

How many counterexamples to (*) are there in the range $0 < n < 50$

- A 2
B 3
C 4
D 5
E 6

15) A cubic polynomial is given by $f(x) = x^3 + bx^2 + cx + d$ where b, c and d are constants.

Two of its factors are $(x - 1)$ and $(x + 1)$

Which of the following statements, taken independently, is/are **necessarily** true?

- I If $f(0) = k$ then $f(k) = 0$
II $f(x) = x^3 - x$
III The graph of $f(x)$ is symmetrical in the y-axis.
- A none of them
B I only
C II only
D III only
E I and II only
F II and III only
G I and III only
H I, II and III

16) Consider the following statements about the polynomial $p(x)$ where a is a constant

- I $p(a) = 0$ and $p'(a) = 0$.
- II $p'(a) = 0$ and $p''(a) > 0$
- III $p''(a) < 0$ and $p(a) > 0$

Which of these statements are sufficient for $p(x)$ to have a local maximum point at $x = a$?

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

17) Which of the following conditions is sufficient but not necessary for $\frac{x}{|x|} < x$

- I $x > 1$
- II $x > -1$
- III $|x| < 1$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

18) Consider the statement: $f(x) > x$ for all real values of $x > 1$

Which one of the following is a negation of this statement?

- A $f(x) \leq x$ for all real values of $x \leq 1$
- B $f(x) \leq x$ for all real values of $x > 1$
- C $f(x) \leq x$ for at least one real value of $x \leq 1$
- D $f(x) \leq x$ for at least one real value of $x > 1$
- E $f(x) > x$ for at least one real value of $x \leq 1$
- F $f(x) > x$ for at least one real value of $x > 1$
- G $f(x) > x$ for no real values of $x \leq 1$
- H $f(x) \leq x$ for no real values of $x > 1$

- 19) A set of cards has a single letter on the front and a single number on the back. Five of these cards are laid on the table so that only one side of each card is visible. The cards show the following: Q K 3 6 8. Dan states that all cards with an Q on one side have an even number on the reverse. Which cards do you need to turn over in order to check his statement?

- A card Q only
- B cards Q and 3
- C cards Q and 6 and 8
- D cards K and 3
- E all of the cards

- 20) Consider the following statement:

If $f'(x) > 0$ for all real x then $f(x + 1) > f(x)$ for all real x

Which function provides a counterexample:

- A $f(x) = 4^x$
- B $f(x) = 4x^2 + 1$
- C $f(x) = 4x^3$
- D $f(x) = \frac{4-x}{x}$
- E $f(x) = \frac{x-1}{4x}$

- 21) x and y are non-zero real numbers. Consider the three statements below:

- I $x > y$ **if** $\frac{x}{y} > 1$
- II $\frac{x}{y} > 1$ **if and only if** $\frac{y}{x} < 1$
- III **If** $xy < 1$ **then** both $x < 1$ and $y < 1$

Which of these statements, taken independently, is/are true?

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F II and III only
- G I and III only
- H I, II and III

- 22) Jill, Kate and Lara each wear a hat. There are three hats: one black, one red, and one blue. It is known that:
- 1) If Jill wears black, then Kate wears blue.
 - 2) If Jill wears red, then Lara wears blue.
 - 3) If Kate does not wear red, then Lara wears black.

What is the colour of the hat Kate is wearing?

- A black
- B red
- C blue
- D there is insufficient information to answer the question

- 23) Consider the following statement:
If $f(x) > 0$ for all $x \geq 0$, then $f'(x) > 0$ for all $x \geq 0$

Which function provides a counterexample:

- | | | | |
|---|-----------------------|---|-----------------------|
| A | $f(x) = x^2 + 3x + 4$ | B | $f(x) = x^2 - 3x + 4$ |
| C | $f(x) = x^2 + 3x - 4$ | D | $f(x) = x^2 - 3x - 4$ |

- 24) If $f(x) = \frac{x}{x+1}$ for all integers $x \neq -1$, which of the following must be true?
- I $f(x+1) > f(x)$ II $f(x) > 0$ III $f(x) \neq 0$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

- 25) A set P of integers is called a *closed set under addition* **if and only if** for any integer a in set P , *there exists* an integer k in P , such that k is the sum of a and b for *all* integers b which are in P .

Which of the following is true **if and only if** P is **not** closed under addition?

- A There exists an integer a in P , such that for any integer k in P , and any integer b in P , k is not the sum of a and b .
- B There exists an integer a in P , and an integer k in P , for which there is no integer b in P , such that k is the sum of a and b .
- C There exists an integer a in P , such that for any integer k in P , there is no integer b in P , such that k is the sum of a and b .
- D There exists an integer a in P , such that for any integer k in P , there is an integer b in P , such that k is not the sum of a and b .
- E For any integer a in P , there exists an integer k in P , and an integer b in P , such that k is not the sum of a and b .
- F For any integer a in P , there exists an integer k in P , and an integer b in P , such that k is the sum of a and b .
- G For any integer a in P , there exists an integer k in P , such that for any integer b in P , k is not the sum of a and b .
- H For any integer a in P , and any integer k in P , there is no integer b in P , such that k is the sum of a and b .