TMUA Multiple Choice Practice - Proof and Logic

1) If $f(x)$ satisfies $f(x) = f(x^2)$ for all real x, which of the following must be true:

A $f(4) = f(2)f(2)$

B $f(16) - f(-2) = 0$

C $f(0) = 0$

D $f(-2) + f(4) = 0$

2) If $a < 0$ and $b < c$, which of the following must be true?

A $ab < c$

B $ac > b$

C $ab > 0$

D $ac < 0$

E $ab > ac$

3) Given that p and q are integers, and that three of the following statements are true, which is the false statement?

A pq is even

B $p + q$ is even

C $2p + q^2$ is odd

D $p^2 + 2q$ is odd

4) Consider the following conjecture:

"If N is a positive integer that consists of the digit 2 followed by an even number of 1 digits, then N is a prime number"

Here are three numbers:

I N = 21 (which equals 3x7)

II N = 211 (a prime number)

III N = 21111 (which equals 227 x 93)

Which of these provide(s) a counterexample to the original conjecture?

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

5) Which of the following is a necessary and sufficient condition for

$$\sum_{k=1}^{n} tan(\frac{k\pi}{3}) = 0$$

A n = 3

B n is a multiple of 3

C n is a multiple of 6

D n is a multiple of 3 or n is 1 less than a multiple of 3

E n is a multiple of 6 or n is 1 less than a multiple of 6

6) The real numbers *a, b, c, d* satisfy

$0 < a + b < c + d$ and $0 < a + c < b + d$

Which of the following must be true:

I $a < d$

II $b < c$

III $a + b + c + d > 0$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

7) The arithmetic mean of five consecutive integers is an odd integer.

Which of the following must be true?

I The largest of the integers is even

II The sum of the integers is odd

III The difference between the largest and smallest of the integers is even.

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

8) Consider the following statement:

If $f(f(x)) = x$ **then** $f(x) = x$

Which function provides a counterexample:

A $f(x) = 1$ B $f(x) = x$ C $f(x) = \dfrac{1}{x}$ D $f(x) = x^2$

9) If *a, b* and *c* are integers, consider the statement $\dfrac{ab^2}{c}$ is a positive even integer (*)

Which of the following is a necessary but not sufficient condition for (*) to be true

I ab is even II $ab > 0$ III c is even

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

13) If $\left|\frac{x}{4}\right| > 1$ which of the following must be true?

I $\quad x > 4$ $\qquad$ II $\quad x \neq 4$ $\qquad$ III $\quad x < -4$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

14) Consider the statement:.

(*) A whole number *n* is prime if it is 1 less or 5 less than a multiple of 8.

How many counterexamples to (*) are there in the range $0 < n < 50$

A 2

B 3

C 4

D 5

E 6

15) A cubic polynomial is given by $f(x) = x^3 + bx^2 + cx + d$ where b, c and d are constants.

Two of its factors are $(x - 1)$ and $(x + 1)$

Which of the following statements, taken independently, is/are **necessarily** true?

I $\quad$ If $f(0) = k$ then $f(k) = 0$

II $\quad f(x) = x^3 - x$

III $\quad$ The graph of f(x) is symmetrical in the y-axis.

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

16) Consider the following statements about the polynomial $p(x)$ where a is a constant

I $\quad p(a) = 0$ and $p'(a) = 0.$

II $\quad p'(a) = 0$ and $p''(a) > 0$

III $\quad p''(a) < 0$ and $p(a) > 0$

Which of these statements are sufficient for $p(x)$ to have a local maximum point at $x = a$?

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

17) Which of the following conditions is sufficient but not necessary for $\dfrac{x}{|x|} < x$

I $\quad x > 1$ $\qquad$ II $\quad x > -1$ $\qquad$ III $\quad |x| < 1$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

18) Consider the statement: $\quad f(x) > x$ for all real values of $x > 1$

Which one of the following is a negation of this statement?

A $\quad f(x) \le x$ for all real values of $x \le 1$
B $\quad f(x) \le x$ for all real values of $x > 1$
C $\quad f(x) \le x$ for at least one real value of $x \le 1$
D $\quad f(x) \le x$ for at least one real value of $x > 1$
E $\quad f(x) > x$ for at least one real value of $x \le 1$
F $\quad f(x) > x$ for at least one real value of $x > 1$
G $\quad f(x) > x$ for no real values of $x \le 1$
H $\quad f(x) \le x$ for no real values of $x > 1$

19) A set of cards has a single letter on the front and a single number on the back.

Five of these cards are laid on the table so that only one side of each card is visible.

The cards show the following: Q K 3 6 8

Dan states that all cards with an Q on one side have an even number on the reverse.

Which cards do you need to turn over in order to check his statement?

A card Q only

B cards Q and 3

C cards Q and 6 and 8

D cards K and 3

E all of the cards

20) Consider the following statement:

If $f'(x) > 0$ *for all real x* *then* $f(x+1) > f(x)$ *for all real x*

Which function provides a counterexample:

A $f(x) = 4^x$ B $f(x) = 4x^2 + 1$ C $f(x) = 4x^3$

D $f(x) = \dfrac{4-x}{x}$ E $f(x) = \dfrac{x-1}{4x}$

21) x and y are non-zero real numbers. Consider the three statements below:

I $x > y$ **if** $\dfrac{x}{y} > 1$

II $\dfrac{x}{y} > 1$ **if and only if** $\dfrac{y}{x} < 1$

III **If** $xy < 1$ **then** both $x < 1$ and $y < 1$

Which of these statements, taken independently, is/are true?

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

22) Jill, Kate and Lara each wear a hat. There are three hats: one black, one red, and one blue.

It is known that:

1) If Jill wears black, then Kate wears blue.

2) If Jill wears red, then Lara wears blue.

3) If Kate does not wear red, then Lara wears black.

What is the colour of the hat Kate is wearing?

A black

B red

C blue

D there is insufficient information to answer the question

23) Consider the following statement:

If $f(x) > 0$ for all $x \geq 0$, **then** $f'(x) > 0$ for all $x \geq 0$

Which function provides a counterexample:

A	$f(x) = x^2 + 3x + 4$	B	$f(x) = x^2 - 3x + 4$
C	$f(x) = x^2 + 3x - 4$	D	$f(x) = x^2 - 3x - 4$

24) If $f(x) = \dfrac{x}{x+1}$ for all integers $x \neq -1$, which of the following must be true?

I $f(x+1) > f(x)$ II $f(x) > 0$ III $f(x) \neq 0$

None / I only / II only / III only / I and II only / I and III only / II and III only / I II and III

25) A set P of integers is called a *closed set under addition* **if and only if** for any integer a in set P, *there exists* an integer k in P, such that k is the sum of a and b *for all* integers b *which are in* P.

Which of the following is true **if and only if** P is **not** *closed under addition*?

A There exists an integer a in P, such that for any integer k in P, and any integer b in P, k is not the sum of a and b.

B There exists an integer a in P, and an integer k in P, for which there is no integer b in P, such that k is the sum of a and b.

C There exists an integer a in P, such that for any integer k in P, there is no integer b in P, such that k is the sum of a and b.

D There exists an integer a in P, such that for any integer k in P, there is an integer b in P, such that k is not the sum of a and b.

E For any integer a in P, there exists an integer k in P, and an integer b in P, such that k is not the sum of a and b.

F For any integer a in P, there exists an integer k in P, and an integer b in P, such that k is the sum of a and b.

G For any integer a in P, there exists an integer k in P, such that for any integer b in P, k is not the sum of a and b.

H For any integer a in P, and any integer k in P, there is no integer b in P, such that k is the sum of a and b.

Quantifiers

These are equivalent:

- For every *A* / for any *A* / for all *A* / for each *A* / if *A*
- For some *A* / there exists *A* / for at least one *A*

The order of a combined statement is important.

For all positive real x, **there exists** a real y such that $y^2 = x$	TRUE (pick any $x>0$)
There exists a real y, such that for all positive real x, $y^2 = x$	FALSE (value of y changes with x)

Original statement	Negation
For all A, then B	*Not every A implies B* *There exists A such that not B*
Every integer is odd	*Not every integer is odd* *There exists an integer that is not odd*

Original statement	Negation
There exists A such that B	*There is no A such that B* *For all A, not B*
There exists a prime number less than 2	*There are no prime numbers less than 2* *All prime numbers are greater than or equal to 2*

<u>Example of negation of 'nested' statements</u>

Statement	*Negated Statement*
There exists P iff for every Q there exists R	*Not P = there exists Q such that 'not R'*
For any D there exists 'E such that F'	*There exists D such that NOT '**E such that F**'* *There exists D such that for all E not F*