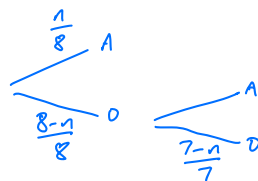


1.

Max has a fruit bowl containing apples and oranges only. There are eight pieces of fruit in total in the bowl. Max chooses two pieces of fruit from the bowl at random and eats them.

The probability of Max not eating any apples is  $\frac{5}{14}$ .

What is the probability that Max eats two apples?



- A  $\frac{3}{28}$       B  $\frac{4}{14}$       C  $\frac{9}{28}$       D  $\frac{3}{7}$       E  $\frac{9}{14}$

$$\begin{aligned} \frac{(8-n)}{8} \times \frac{(7-n)}{7} &= \frac{5}{14} \\ 56 - 15n + n^2 &= 20 \\ n^2 - 15n + 36 &= 0 \end{aligned}$$

$$\begin{aligned} (n-3)(n-12) &= 0 \\ n=3 \quad (n \neq 12) \end{aligned}$$

$$P(A.A) = \frac{2}{8} \times \frac{2}{7} = \frac{3}{28}$$

2.

In a set of  $k$  consecutive integers, the largest number is 39. What is the mean of the set?

- A  $\frac{k+38}{k}$   
B  $41-2k$   
C  $\frac{40-k}{k}$   
D  $\frac{79-k}{2}$   
E  $\frac{k+77}{2}$

$$39-k+1 \dots \dots 39$$

$$\begin{aligned} \text{Sum} &= \frac{k}{2} (39-k+1 + 39) \\ &= \frac{k}{2} (79-k) \end{aligned}$$

$$\text{Mean} = \frac{1}{2} (79-k)$$

3.

A bag contains five different coloured balls. Two players take it in turns to draw a ball out of the bag. If the ball is red, they win the game. If the ball is not red, they replace the ball in the bag and the other player takes their turn.

What is the probability that the first player to choose a ball does **not** win the game?

- A  $\frac{4}{9}$       B  $\frac{1}{2}$       C  $\frac{6}{11}$       D  $\frac{5}{9}$       E  $\frac{4}{7}$

$$P(\text{1st does not win}) = \left(\frac{4}{5} \times \frac{1}{5}\right) + \left(\frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}\right) + \dots$$

$$\text{C.P. } a = \frac{4}{25} \quad r = \frac{16}{25} \quad S_{\infty} = \frac{4/25}{1-16/25} = \frac{4}{9}$$

4.

A box of chocolates contains an assortment of milk, dark and white chocolates. Three friends each take a chocolate from the box. Consider the following probabilities:

$J = P(\text{the last person chooses a white chocolate})$

$K = P(\text{at least one person chooses a white chocolate})$  *largest*

$L = P(\text{exactly two people choose a white chocolate})$  *smallest*

Which of the following is true?

A  $J < K < L$

B  $J < L < K$

C  $K < J < L$

D  $K < L < J$

**E**  $L < J < K$

F  $L < K < J$

5.

A list of five positive integers has a mean of 40 and a range of 10.

What is the maximum possible value of the greatest integer in the list?

A 44

**B** 48

C 49

D 50

E 52

$a \quad - \quad - \quad - \quad a + 10$  *sum = 200*  
 $\uparrow$   
 want max value -  
 this occurs when first 4 terms are  
 as small as possible  
 eg  $a \quad a \quad a \quad a \quad a + 10$   
 $\therefore 5a + 10 = 200$   
 $a = 38$   
 $a + 10 = 48$

6.

A bag contains different coloured balls, some of which are red. Alex removes 2 balls from the bag.

Given that the probability that exactly one of these balls is red, is the same as the probability that neither of these balls is red, how many balls in total could be in the bag?

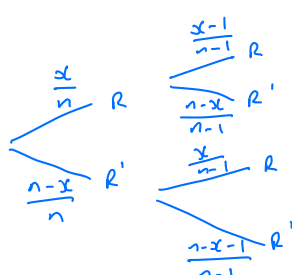
A 8

B 9

**C** 10

D 11

E 12



$P(\text{exactly 1 red}) = P(\text{no reds})$   
 $\frac{2x(n-x)}{n(n-1)} = \frac{(n-x)(n-x-1)}{n(n-1)}$   
 $n \neq x$   $2x = n - x - 1$   
 $n = 3x + 1$  *(one more than a multiple of 3)*

7.

There are red balls and green balls in a bag. There are more green balls than red balls.  $g > r$

Rhea picks a ball from the bag and records the colour. She then replaces the ball in the bag. She repeats the process, recording the colour of the second ball.

Given that  $P(2 \text{ green balls}) - P(2 \text{ red balls}) = \frac{3}{5}$

$$g^2 - r^2 = \frac{3}{5}$$

$$(g-r)(g+r) = \frac{3}{5}$$

$$g+r = 1$$

$$g-r = \frac{3}{5}$$

$$2g = \frac{8}{5}$$

$$g = \frac{4}{5} \quad r = \frac{1}{5}$$

what is the probability of Rhea choosing one ball of each colour?

- A  $\frac{4}{25}$       B  $\frac{1}{5}$       C  $\frac{6}{25}$       **D  $\frac{8}{25}$**       E  $\frac{2}{5}$

$$P(\text{one of each}) = P(r, g) + P(g, r) = 2rg = 2 \times \frac{1}{5} \times \frac{4}{5} = \frac{8}{25}$$

8.

A class of 25 students take a maths test and their mean mark is calculated to be 60. The range of their marks is 40.

Five new students join the class and take the same test. The mean mark for the whole class of 30 students is now calculated to be 55.

Which of the following statements are necessarily true?

- I All of the five new students score less than 60.  $\times$  *no: could have 1 above 60 and others much lower eg 65, 20, 20, 20, 25*
- II The mean mark for just the five new students is 50.  $\times$
- III The range of the marks for the whole class of 30 students is **not** 40.  $\times$  *could be eg initial scores 30...70 new students all score 30*

- A** none of them
- B I only
- C II only
- D III only
- E I and II only
- F II and III only
- G I and III only
- H I, II and III

$$25 \times 60 = 1500 \quad \text{total for 25 students}$$

$$30 \times 55 = 1650 \quad \text{total for 30 students}$$

$$\text{mean for 5 students} = \frac{1650 - 1500}{5} = 30$$

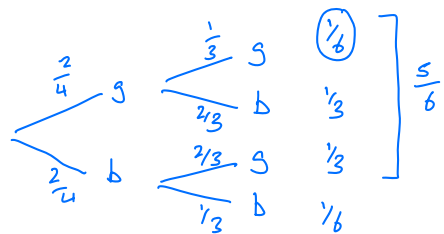
9.

There are two green socks and two blue socks in a drawer.

Two socks are removed at random without replacement.

Given that at least one of them is green, what is the probability that both of them are green?

$$P(g, g \mid \text{at least 1 } g) = \frac{1/6}{5/6} = \frac{1}{5}$$



- A  $\frac{1}{6}$       **B**  $\frac{1}{5}$       C  $\frac{1}{4}$       D  $\frac{2}{5}$       E  $\frac{1}{2}$

10.

The ratio of the number of boys to the number of girls in a class is 2:3.

Two students are chosen at random from the class.

The probability that both the students are boys is  $p$ .

Which of the following is an expression for the number of boys in the class.  $= 2n$

A  $\frac{5p-2}{25p-4}$

B  $\frac{5p+2}{25p-4}$

**C**  $\frac{10p-4}{25p-4}$

D  $\frac{p-2}{9p-4}$

E  $\frac{5p-2}{9p-4}$

$$P(2 \text{ boys}) = \frac{2}{5} \times \frac{2n-1}{5n-1} = p$$

$$4n-2 = 25np - 5p$$

$$5p-2 = 25np - 4n$$

$$n = \frac{5p-2}{25p-4}$$

$$2n = \frac{10p-4}{25p-4}$$

11.

When the integer 22 is added to a list of integers, the mean increases by 2.

When the integer 15 is added to the new list, the mean increases again by a further 1.

How many integers were in the original list?

- A 5      **B** 9      C 10      D 12      E 15

original  $k$  integers with mean  $m$   
 $\therefore$  sum is  $mk$

$$\frac{mk+22}{k+1} = m+2$$

$$22 = 2k + m + 2$$

$$40 = 4k + 2m \quad (1)$$

$$\frac{mk+37}{k+2} = m+3$$

$$37 = 2m + 3k + 1$$

$$31 = 3k + 2m \quad (2)$$

$$(1) - (2) \quad k = 9$$

12.

$$\text{Total : 8 tails + 12 heads} \quad P(H) = \frac{12}{20}$$

I have a bag of ten coins. Eight of the coins are ordinary fair coins with a head on one side and a tail on the other. The final two coins are double headed, so they have a head on each side.

A single coin is removed from the bag and placed on a table. Given that this coin shows a head, what is the probability that it is a double headed coin?

A  $\frac{1}{5}$

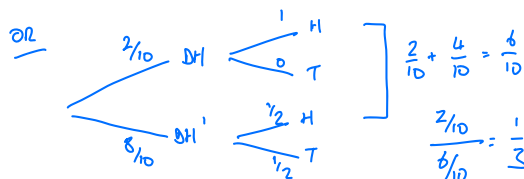
(B)  $\frac{1}{3}$

C  $\frac{2}{5}$

D  $\frac{1}{2}$

E  $\frac{3}{5}$

$$P(\text{double headed} | \text{head}) = \frac{2/10}{12/20} = \frac{1}{3}$$



13.

A chord is drawn at random on a circle of radius 10cm. What is the probability that the length of the chord is less than 10cm?

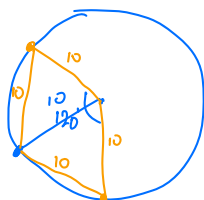
A  $\frac{1}{2\pi}$

B  $\frac{1}{6}$

C  $\frac{1}{\pi}$

(D)  $\frac{1}{3}$

E  $\frac{\pi}{6}$



fix point on circumference  
 2nd point must have chord < 10cm in either direction  
 Equilateral triangles  $\Rightarrow$  angle at centre is  $120^\circ$  ( $= \frac{1}{3}$  of circle)  
 or  $\frac{\text{length of arc}}{\text{circumference}} = \frac{\frac{1}{3} \times 2\pi \times 10^2}{2\pi \times 10^2} = \frac{1}{3}$

14.

A TMUA paper consists of 20 multiple choice questions, each having five possible answers.

A student believes that there is a probability of 0.65 that they know the answer to any question. If they do not know the answer they will guess the answer with probability  $\frac{1}{5}$  of being correct.

What is the probability that the student will give the correct answer to a question?

A 0.65

B 0.69

(C) 0.72

D 0.75

E 0.85

$$\begin{aligned} P(\text{correct}) &= 0.65 + (0.35 \times 0.2) \\ &= 0.65 + 0.07 \\ &= 0.72 \end{aligned}$$

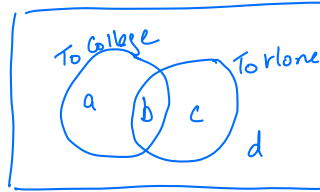
15.

On any given day the probability that Ray will walk to college is more than 0.6. Also the probability that Ray will walk to college and walk home again is less than 0.5.

The probability that Ray will walk to college or walk home is equal to 0.7.

If  $p$  is the probability that Ray will walk home, what is the range of possible values for  $p$ ?

- A  $0 < p < 0.6$
- B  $0 \leq p < 0.6$**
- C  $0 \leq p \leq 0.6$
- D  $0 < p < \frac{5}{6}$
- E  $0 \leq p < \frac{5}{6}$
- F  $0 \leq p \leq \frac{5}{6}$



$$a+b > 0.6$$

$$b < 0.5$$

$$a+b+c = 0.7$$

$$p = b+c$$

$$a+b > 0.6$$

$$0.7 - c > 0.6$$

$$c < 0.1$$

$$b < 0.5$$

$$b+c < 0.6$$

so option A or B

$$c - p = 0?$$

$$b = -c \quad a = 0.7$$

$$b = c = 0$$

Yes

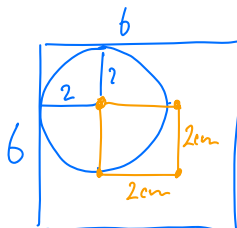
16.

$$r=2$$

A coin of diameter 4cm is dropped onto a floor which is marked with squares of side length 6cm (like a giant chess board).

Given that the coin lands away from the edge of the floor, what is the probability that the coin lies completely inside a single square.

- A  $\frac{1}{9}$**
- B  $\frac{1}{3}$
- C  $\frac{\pi}{9}$
- D  $\frac{2}{5}$
- E  $\frac{\pi}{4}$



Centre of circle must be  $> 2\text{cm}$  from side of square.

$\therefore$  Must lie inside a smaller square of side 2cm

$$P = \frac{2 \times 2}{6 \times 6} = \frac{1}{9}$$