

TMUA Practice - Graphs of Functions

1. Given that $f(x) = x^2 - 5x + 7 = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{28}{4} = \left(x - \frac{5}{2}\right)^2 + \frac{3}{4}$

Find the sum of the x - and y - coordinates of the minimum point of $y = f(x - 2)$

translation 2 units right

- A $\frac{21}{4}$ B $\frac{13}{4}$ C $\frac{5}{4}$ D $\frac{1}{4}$ E $-\frac{7}{4}$

$\text{Min } f(x) = \left(\frac{5}{2}, \frac{3}{4}\right)$

$f(x-2) = \left(\frac{9}{2}, \frac{3}{4}\right)$

$\frac{9}{2} + \frac{3}{4} = \frac{21}{4}$

2. The curve with equation $x^9 + x^7 + y^4 + y^8 = 2$ has

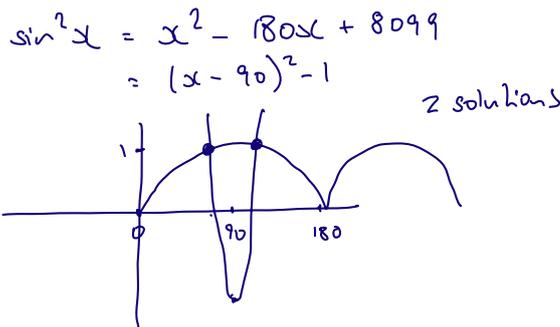
- A neither the x -axis nor y -axis as a line of symmetry
 B the x -axis but not the y -axis as a line of symmetry
 C the y -axis but not the x -axis as a line of symmetry
 D both axes as lines of symmetry

$y \leftrightarrow -y$ same, as even powers of y
 $x \not\leftrightarrow -x$ not same, as odd powers of x

3. How many solutions does the following equation have (where x is given in degrees)?

$\sin^2 x = x^2 - 180x + 8099$

- A 0
 B 1
 C 2
 D 4
 E 8
 F infinitely many



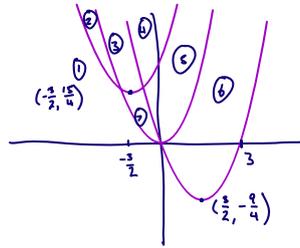
4. How many regions are there, (excluding the coordinate axes) when the following curves are drawn?

$$y = x^2$$

$$y = x^2 - 3x = x(x-3)$$

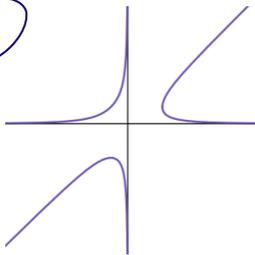
$$y = x^2 + 3x + 6 = \left(x + \frac{3}{2}\right)^2 + \frac{15}{4}$$

- A 4
B 5
C 6
D 7
E 8

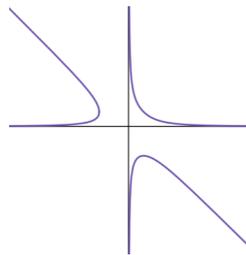


5. A sketch of the curve with equation $xy(x - y) = 1$ is drawn in:

A



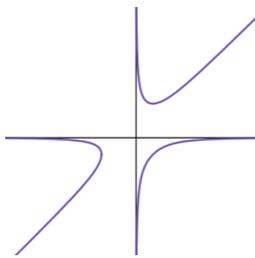
B



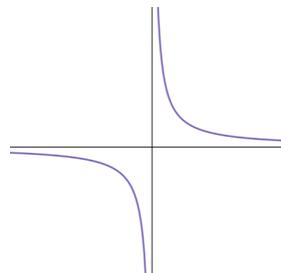
asymptotes
 $x=0, y=0, x=y$
so not B, D

$x > 0, y < 0$
 $x - y > 0$
 $xy < 0$ X
so not C

C



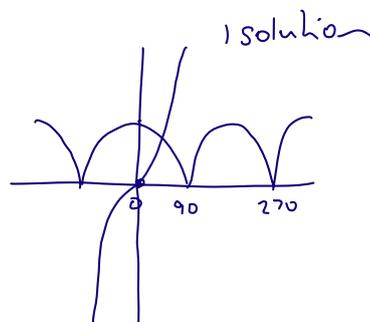
D



6. How many solutions does the following equation have

$$\cos^2 x = x^3$$

- A 0
B 1
C 2
D 4
E 8
F infinitely many



7. The graph of $y = 2^{x^2}$ has a series of transformations applied, resulting in the graph of $y = 2^{x^2+2x+4}$

Which of the following could be the sequence of transformations?

- A a translation parallel to the x -axis, followed by a stretch parallel to the y -axis
- B a translation parallel to the x -axis, followed by a translation parallel to the y -axis
- C a translation parallel to the y -axis, followed by a stretch parallel to the y -axis
- D a stretch parallel to the x -axis, followed by a translation parallel to the x -axis
- E a stretch parallel to the x -axis, followed by a translation parallel to the y -axis
- F a stretch parallel to the x -axis, followed by a stretch parallel to the y -axis

$$y = 2^{(x+1)^2+3} = 2^3 \cdot 2^{(x+1)^2}$$

$2^{x^2} \rightarrow$ translation 1 unit left x direction $2^{(x+1)^2} \rightarrow$ stretch SF $2^3=8$ // by y axis

8. The graph of $y = f(x)$ intersects the x -axis at exactly two distinct points.

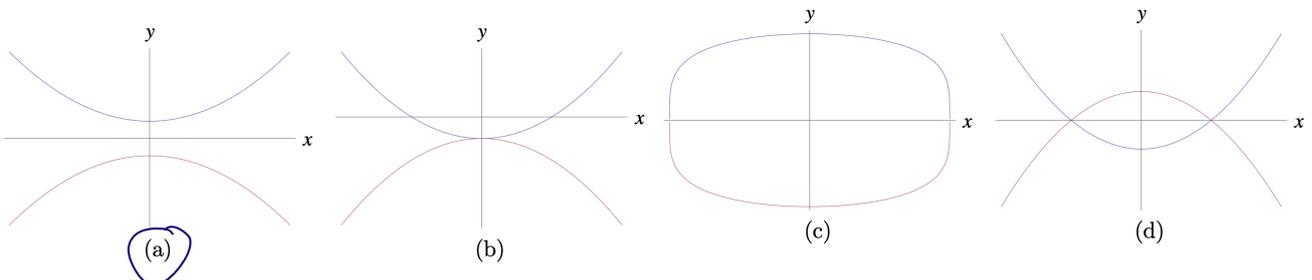
Consider the following five graphs:

$y = f(x) - 3$ translation down	$y = f(x - 3)$ translation right	$y = 3f(x)$ stretch by y	$y = 3 - f(x)$ reflection in x -axis and translation up	$y = f(-3x)$ reflection in y -axis and stretch in x
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How many of these graphs necessarily intersect the x -axis at exactly two distinct points?

- A 0
- B 1
- C 2
- D 3
- E 4
- F 5

9. Which of the following is a sketch of $y^2 - x^4 = 4$



$$x^4 = y^2 - 4$$

$$x = 0 \quad y = \pm 2$$

$$x^4 \geq 0 \quad y^2 \geq 4$$

$y \geq 2, y \leq -2$ so no solutions $-2 < y < 2$

10. The graph of a quadratic curve has equation $y = a + bx - x^2$

The image of the curve when reflected in the y-axis is identical to the image of the curve when translated 3 units in the negative x-direction. What is the value of b ?

- A $b = -3$
- B $b = -1$
- C $b = 1$
- D** $b = 3$
- E $b = 9$

$$f(-x) = f(x+3)$$

$$a - bx - x^2 = a + bx + 3b - (x+3)^2$$

$$0 = 2bx + 3b - 6x - 9$$

$$0 = (2b-6)x + (3b-9)$$

$$b = 3$$

11. A sketch of the curve with equation $y = 1 - |1 - |x||$ is drawn in:

A

B

C

D

$|x|$

$-|x|$

$1 - |x|$

$|1 - |x||$

$-|1 - |x||$

$1 - |1 - |x||$

12. The function f is such that $f(x) = \frac{x-k}{x^2-4x-k}$, $x \in \mathbb{R}$ where k is a constant, and $x^2 - 4x - k \neq 0$

Given that the **range** of $f(x)$ is all the real numbers, what are the possible values of k ?

- A $-5 \leq k \leq 5$
- B $k \leq -5, k \geq 5$
- C** $0 \leq k \leq 5$
- D $k \leq 0, k \geq 5$
- E $k = 0$ or $k = 5$

Let $y = \frac{x-k}{x^2-4x-k}$

$$x^2y - 4xy - ky = x - k$$

$$x^2y - (4y+1)x + (k-ky) = 0$$

quadratic in x must have $\Delta \geq 0$
in order that there is always a solution

$$\Delta = (4y+1)^2 - 4y(k-ky) \geq 0$$

$$16y^2 + 8y + 1 - 4ky + 4ky^2 \geq 0$$

$$(16+4k)y^2 + (8-4k)y + 1 \geq 0$$

quadratic in $y \geq 0$ must have $\Delta \leq 0$

$$\Delta = (8-4k)^2 - 4(16+4k)(1) \leq 0$$

$$(2-k)^2 - 4 - k \leq 0$$

$$4 - 4k + k^2 - 4 - k \leq 0$$

$$k^2 - 5k \leq 0$$

$$k(k-5) \leq 0$$

$$0 \leq k \leq 5$$

13. Consider the following function $f(x) = \frac{x^2 + 3x + 2}{x + 4} = x - 1 + \frac{6}{x + 4}$

What can be said about the asymptote(s) of the graph of this function?

- A The graph has an asymptote at $x = -4$
- B** The graph has asymptotes at $x = -4$ and at $y = x - 1$
- C The graph has asymptotes at $x = -4$ and at $y = x$
- D The graph has asymptotes at $x = 0$ and at $y = x$
- E The graph has asymptotes at $x = 0$ and at $y = \frac{1}{2}$

$x \rightarrow \infty$
 $f(x) \rightarrow x - 1$

$x^2 + 3x + 2 = (x + 4)(x - 1) + 6$

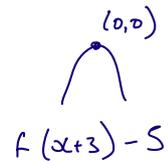
14. The graph of a quadratic function $f(x)$ has a maximum point at $(3, 5)$

The graph $y = f(x)$ is transformed onto the graph of $y = g(x)$ so that the graph of $g(x)$ has a minimum point at the origin. What is the equation for $g(x)$?

- A $5 - f(x - 3)$
- B** $5 - f(x + 3)$
- C $5 - f(3 - x)$
- D $f(x + 3) - 5$
- E $f(3 - x) - 5$



Translate left 3
down 5

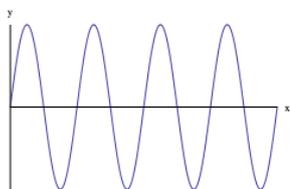


Reflect in x-axis
 $- [f(x+3) - 5]$
 $5 - f(x+3)$

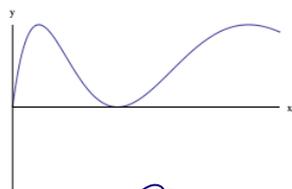


15. Which is the graph of $y = \sin^2 \sqrt{x}$

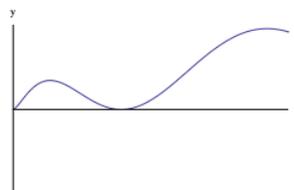
$y > 0$ $(0, 0)$ $y = 0$ $\sqrt{x} = 0, \pi, 2\pi$
 $x = 0, \pi^2, 4\pi^2, 9\pi^2$



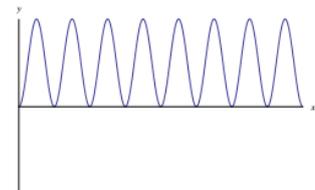
(a)
 $x, y > 0$



(b)



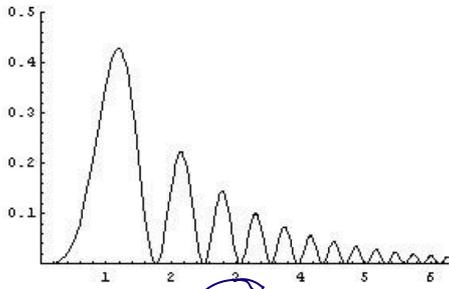
(c)
 x peaks not same



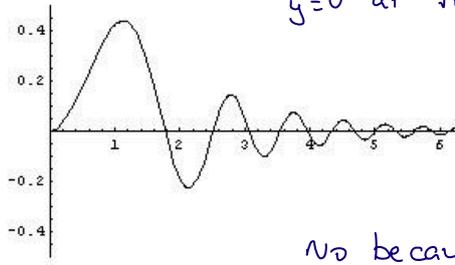
(d)
 x roots too regular

16. Which of the following is a sketch of the graph $y = 2^{-x} \sin^2(x^2)$

$y > 0$ $y(0) = 0$
 $y = 0$ at $\sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}$

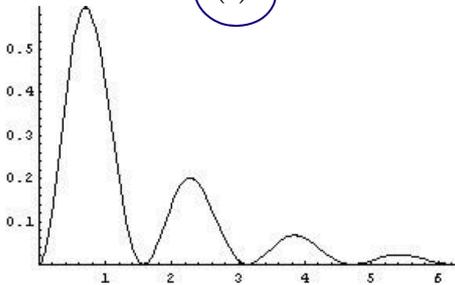


(a)



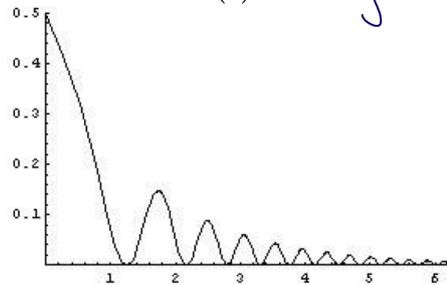
(b)

No because $y > 0$



(c)

roots too regular

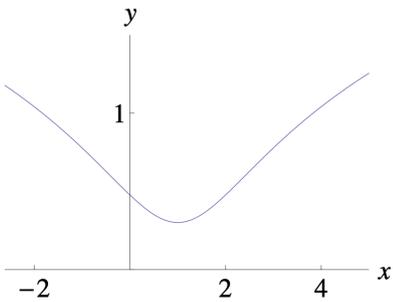


(d)

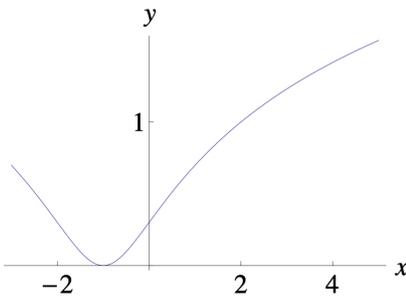
not (0,0)

17. Which of the following is a sketch of

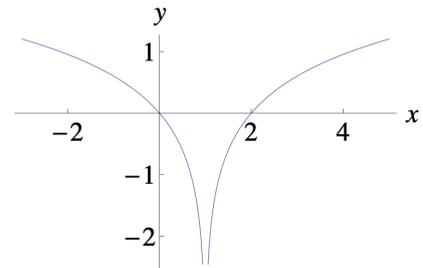
$y = \log_{10}(x^2 - 2x + 2) = \log_{10}[(x-1)^2 + 1]$ valid for all x
 min at $x=1$ (1,0)



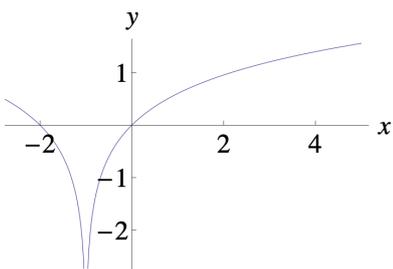
(a)



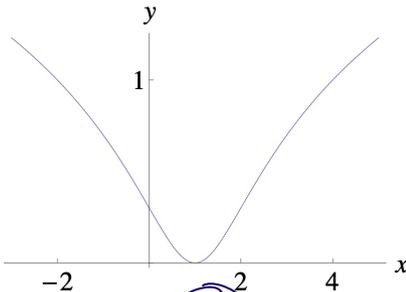
(b)



(c)



(d)



(e)