

1. What is the largest solution for x in the range $0 \leq x < 2\pi$ for the following equation:



$$2\sin\left(2x - \frac{\pi}{3}\right) + 1 = 0 \quad -\frac{\pi}{3} \leq 2x - \frac{\pi}{3} \leq \frac{\pi}{3}$$

- A $\frac{\pi}{12}$ B $\frac{3\pi}{4}$ C $\frac{13\pi}{12}$ **D $\frac{7\pi}{4}$** E $\frac{23\pi}{12}$

$$\sin\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}$$

$$2x - \frac{\pi}{3} = \frac{7\pi}{6} \quad \frac{11\pi}{6}$$

largest
↓
 $\frac{19\pi}{6}$ $\frac{23\pi}{6}$
out of range

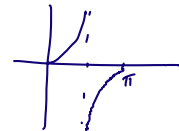
$$2x = \frac{19\pi}{6} + \frac{\pi}{3} = \frac{21\pi}{6} = \frac{7\pi}{2}$$

$$x = \frac{7\pi}{4}$$

2. What is the sum of the solutions for x in the range $0 \leq x < \pi$ for the following equation:

$$\tan(2x - \pi) = 1 \quad -\pi \leq 2x - \pi < \pi$$

- A $\frac{\pi}{8}$ B $\frac{5\pi}{8}$ **C $\frac{3\pi}{4}$** D $\frac{5\pi}{4}$ E $\frac{7\pi}{4}$



$$2x - \pi = -\frac{3\pi}{4} \quad \frac{\pi}{4}$$

$$2x = \frac{\pi}{4} \quad \frac{5\pi}{4}$$

$$x = \frac{\pi}{8} \quad \frac{5\pi}{8}$$

$$\frac{\pi}{8} + \frac{5\pi}{8} = \frac{6\pi}{8} = \frac{3\pi}{4}$$

3. How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$2\sin(\cos x) = \sqrt{2}$$

A 0

B 1

C 2

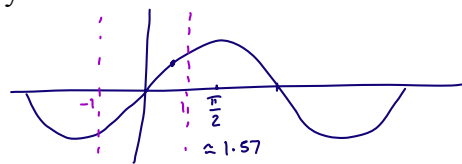
D 3

E infinitely many

$$\text{Let } A = \cos x \quad -1 \leq \cos x < 1$$

$$\text{Solve } 2\sin A = \sqrt{2} \quad \text{for } -1 \leq A < 1$$

$$\sin A = \frac{\sqrt{2}}{2}$$



$$\frac{\pi}{4} < 1$$

$$1 \text{ solution } A = \frac{\pi}{4}$$

$$\cos x = \frac{\pi}{4} \text{ has 2 solutions } 0 \leq x < 2\pi$$

4. x satisfies the simultaneous equations

$$2\sqrt{2}\sin 3x - \tan 3x = 3$$

$$\sqrt{2}\tan 3x + 4\sin 3x = \sqrt{2}$$

where $0 \leq x \leq 180$.

Find the sum of the possible values of x

- A 150
 (B) 210
 C 315
 D 360
 E 540

$$0 \leq 3x \leq 540$$

$$\sin 3x = \frac{1}{2}\sqrt{2} : 3x = 45, 135, 405, 495$$

$$\tan 3x = -1 : 3x = 135, 495$$

$$x = 45, 165$$

$$\text{Sum} = 45 + 165 = 210$$

$$\textcircled{1} + \textcircled{2} \quad 4\sqrt{2}S = 4 \quad S = \frac{\sqrt{2}}{2}$$

$$\textcircled{1} - \textcircled{2} \quad -2T = 2 \quad T = -1$$

$$2\sqrt{2}S - T = 3 \quad \textcircled{1}$$

$$4S + \sqrt{2}T = \sqrt{2}$$

$$\div \sqrt{2}$$

$$2\sqrt{2}S + T = 1 \quad \textcircled{2}$$

$$\sin 3x = \frac{1}{2}\sqrt{2}$$

$$\tan 3x = -1$$

5. Consider the inequality

$$\sin\left(x + \frac{\pi}{3}\right) \geq \frac{1}{2}$$

The fraction of the interval $0 \leq x \leq 2\pi$ for which this is true, is:

A $\frac{1}{6}$

B $\frac{1}{4}$

(C) $\frac{1}{3}$

D $\frac{5}{12}$

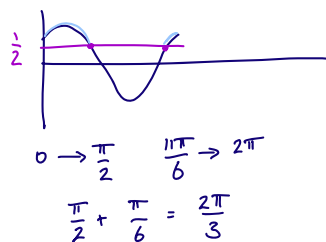
E $\frac{1}{2}$

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$x + \frac{\pi}{3} = \left(\frac{\pi}{6}\right) \quad \frac{5\pi}{6} \quad \frac{13\pi}{6}$$

$$x = \frac{3\pi}{6} \quad \frac{11\pi}{6}$$

$$= \frac{\pi}{2}, \frac{11\pi}{6}$$



$$\frac{2\pi/3}{2\pi} = \frac{1}{3}$$

$$\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{7\pi}{3}$$

OR * Same fraction as $y = \sin x$
 $\Rightarrow \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$

$$\frac{4\pi/6}{2\pi} = \frac{1}{3}$$

6. Find the greatest value of the function $f(x) = (3\sin^2(2x - 5) - 7)^2$

A 16

B 25

C 36

(D) 49

E 100

$$0 \leq \sin^2 A \leq 1$$

$$0 \leq 3\sin^2 A \leq 3$$

$$-7 \leq 3\sin^2 A - 7 \leq 4$$

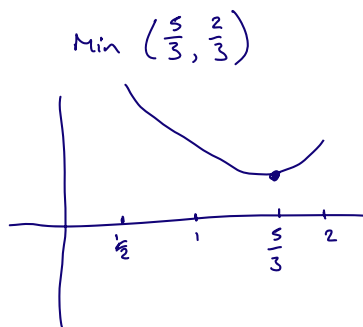
$$\text{Max } (-7)^2 = 49$$

7. Find the maximum value of $3(4^{\sin x}) - 10(2^{\sin x}) + 9$

A $\frac{2}{3}$ B 1 C 2 **D $\frac{19}{4}$** E 9

Let $y = 2^{\sin x}$ $\frac{1}{2} \leq y \leq 2$

$$\begin{aligned} 3y^2 - 10y + 9 \\ 3\left(y^2 - \frac{10}{3}y\right) + 9 \\ 3\left[\left(y - \frac{5}{3}\right)^2 - \frac{25}{9}\right] + 9 \\ 3\left(y - \frac{5}{3}\right)^2 - \frac{25}{3} + \frac{27}{3} \\ 3\left(y - \frac{5}{3}\right)^2 + \frac{2}{3} \end{aligned}$$



Max at $y = \frac{1}{2}$ $\sin x = -1$

$$\begin{aligned} \frac{3}{4} - 5 + 9 &= \frac{19}{4} \\ \text{or} \quad 3\left(\frac{1}{2} - \frac{5}{3}\right)^2 + \frac{2}{3} &= 3\left(-\frac{7}{6}\right)^2 + \frac{2}{3} \\ &= 3\left(\frac{49}{36}\right) + \frac{8}{12} = \frac{57}{12} = \frac{19}{4} \end{aligned}$$

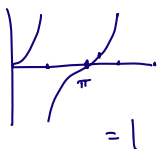
8. Which of the following is the largest?

A $\tan\left(\frac{5\pi}{4}\right)$

B $\sin^2\left(\frac{3\pi}{4}\right)$

C $\log_{10}\left(\frac{5\pi}{4}\right)$

D $\log_2\left(\frac{3\pi}{4}\right)$



$$\begin{aligned} \sin \frac{3\pi}{4} &< 1 \\ \sin^2 \frac{3\pi}{4} &< 1 \\ &\times \\ &< A \end{aligned}$$

$$\begin{aligned} \log_{10}(1.25 \times 3.14) \\ < 1 \\ &\times \\ &< A \end{aligned}$$

$$\begin{aligned} \log_2\left(\frac{1}{4} \times 9 \dots\right) \\ > 1 \\ > A \end{aligned}$$

9. A triangle ABC is drawn with $AC = 5\text{cm}$ and $BC = 11\text{cm}$ and the angle at B equal to a specified angle θ .

Of the two possible triangles that could be drawn, the larger triangle has double the area of the smaller one.

What is the value of $\cos \theta$?

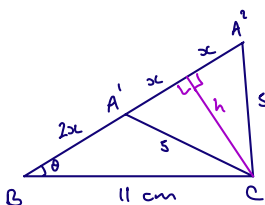
A $\frac{10}{11}$

B $\frac{3\sqrt{12}}{11}$

C $\frac{\sqrt{13}}{11}$

D $\frac{\sqrt{6}}{5}$

E $\frac{3\sqrt{6}}{25}$



$$\begin{aligned} \text{Area} &= \frac{1}{2} AB \times 11 \times \sin \theta \\ \text{Area } A'BC &= 2 \times \text{Area } A'BC \\ A'B &= 2A'B \\ \text{Let } A'B &= 2x \quad A'B = 4x \end{aligned}$$

$$\begin{aligned} x^2 + h^2 &= 25 \\ 9x^2 + h^2 &= 121 \\ 8x^2 &= 96 \\ x^2 &= 12 \\ h^2 &= 13 \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{3x}{11} \\ &= \frac{3\sqrt{12}}{11} \end{aligned}$$

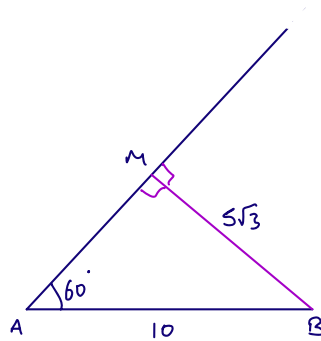
10. A triangle ABC is to be drawn with the following measurements.

$AB = 10\text{cm}$ and angle $BAC = 60^\circ$.

Which of the following statements is/are true ?

- I No such triangle can be drawn if $BC = 7\text{cm}$
- II Exactly one distinct triangle can be drawn if $BC = 5\sqrt{3}\text{cm}$
- III Exactly two distinct triangles can be drawn if $BC = 12\text{cm}$

- A none of them
- B I only
- C II only
- D III only
- E I and II only**
- F II and III only
- G I and III only
- H I, II and III



$$AB = 10 \sin 60^\circ \\ = 5\sqrt{3} \approx 8.5$$

I $7 < 5\sqrt{3}$ true

II true

III $12 > AB$ false
only 1 triangle