Mock TMUA Set C: Paper 1

20 questions

75 minutes

No calculator allowed

1.

The function f is given by $\qquad f(x) = \left(\frac{x}{2} - \frac{6}{x^2}\right)^2 \qquad x \neq 0$

What is the value of $f''(2)$

A $-\frac{9}{2}$ B $\frac{25}{4}$ C 9 D $\frac{41}{4}$ E $\frac{53}{2}$

$f(x) = \frac{x^2}{4} - \frac{6}{x} + \frac{36}{x^4}$

$= \frac{x^2}{4} - 6x^{-1} + 36x^{-4}$

$f'(x) = \frac{1}{2}x + 6x^{-2} - 144x^{-5}$

$f''(x) = \frac{1}{2} - 12x^{-3} + 720x^{-6}$

$f''(2) = \frac{1}{2} - \frac{12}{8} + \frac{720}{64}$

$= \frac{4 - 12 + 90}{8} = \frac{41}{4}$

2.

A line l has equation $3y + x = 15$.

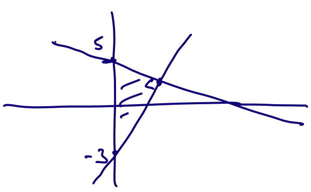
A second line is perpendicular to l and passes through the point $(0, -3)$.

Find the area of the region enclosed by the two lines and the y-axis.

A $4\frac{1}{5}$ B 8 C $9\frac{3}{5}$ D 12 E $19\frac{1}{5}$

$y = -\frac{1}{3}x + 5$

$y = 3x - 3$



$3x - 3 = -\frac{1}{3}x + 5$

$10x = 24$

$x = 2.4$

$\text{Area} = \frac{1}{2} \times 8 \times 2.4 = 9.6$

3.

$f(x)$ is a quadratic function in x.

The graph of $f(x)$ passes through the point $(0,1)$ and has a turning point at $(-1, -1)$.

Find an expression for $f(x)$.

A $f(x) = 2x^2 + 4x + 1$

B $f(x) = 2x^2 + 5x + 2$

C $f(x) = 3x^2 + 5x + 1$

D $f(x) = -2x^2 + 1$

E $f(x) = -x^2 + x + 1$

$f(x) = a(x+1)^2 - 1$

$f(0) = 1 \qquad a - 1 = 1$

$a = 2$

$f(x) = 2(x^2 + 2x + 1) - 1$

$= 2x^2 + 4x + 1$

4.

Given that $\displaystyle\int_1^2 \frac{2a + bx^2}{x^2}\, dx = 2$ find the value of $a + b$

A 0 B $\frac{1}{4}$ C $\frac{1}{2}$ D 1 E 2

$\int_1^2 2ax^{-2} + b \; dx = \left[-2ax^{-1} + bx\right]_1^2 = (-a + 2b) - (-2a + b) = 2$

$a + b = 2$

5.

The 1st, 2nd and 3rd terms of a geometric progression are also the 1st, 4th and 5th terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 9.

Find the first term of the geometric progression.

G.P. $a \quad ar \quad ar^2$

A.P $a \quad a+3d \quad a+4d$

A 4 B $\frac{14}{3}$ C 5 D $\frac{16}{3}$ E 6

$\frac{a}{1-r} = 9$

$a = 9 - 9r$

$r = \frac{9-a}{9}$

$ar = a + 3d$

$ar^2 = a + 4d$

$\frac{ar - a}{3} = \frac{ar^2 - a}{4}$

$a \neq 0 \quad 4r - 4 = 3r^2 - 3$

$3r^2 - 4r + 1 = 0$

$(r-1)(3r-1) = 0$

$|r| < 1 \quad r = \frac{1}{3}$

$a = 9 - 3 = 6$

6.

The two circles with equations below have exactly one point in common.

$(x+2)^2 + (y-3)^2 = 49$ and $(x-4)^2 + (y+5)^2 = r^2$ where $r > 0$

Find the sum of the two possible values of r .

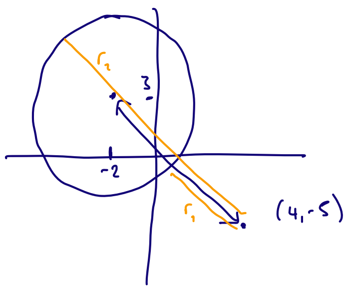
A 17 B 20 C 21 D 24 E 28

Centre $(-2, 3)$ $r = 7$

Centre $(4, -5)$ r

Distance between centres $= \sqrt{6^2 + 8^2}$

$= 10$



$r_1 = 10 - 7 = 3$ +

$r_2 = 10 + 7 = 17$

20

7.

What is the coefficient of x^3 in the series expansion of $(x - \frac{1}{2})^7(x^2 + 1)^3$

$\binom{7}{r} x^r \left(-\frac{1}{2}\right)^{7-r}$

A $\quad \frac{161}{32}$

B $\quad \frac{147}{64}$

C $\quad \frac{161}{64}$

D $\quad \frac{147}{32}$

$(x^6 + 3x^4 + 3x^2 + 1)(x - \frac{1}{2})^7$

$r = 1 \quad 3x^2 \times 7 \times \left(-\frac{1}{2}\right)^6 x = \frac{21}{64}$

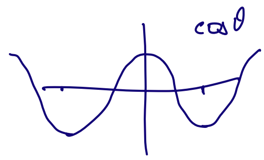
$r = 3 \quad 1 \times \frac{7 \times 6 \times 5}{6} \left(-\frac{1}{2}\right)^4 x^3 = \frac{35}{16} = \frac{140}{64}$

$\frac{161}{64}$

8.

How many solutions are there to

$$(4cos2\theta - 1)^2 = 9 \qquad \text{for } -180^\circ \leq \theta \leq 180^\circ \text{ ?}$$



A $\quad 4$ $\qquad$ B $\quad 5$ $\qquad$ C $\quad 6$ $\qquad$ D $\quad 7$ $\qquad$ E $\quad 8$

$4\cos 2\theta - 1 = \pm 3$

$\cos 2\theta = 1$

$\theta = -360, 0, 360$

3 solutions

$-360 \leq 2\theta \leq 360$

$\cos 2\theta = -\frac{1}{2}$

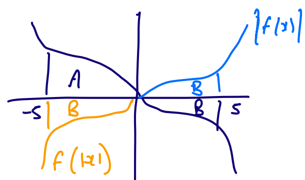
4 solutions

9.

The function $f(x)$ is such that $\quad f(0) = 0 \quad$ and $\quad xf(x) < 0 \quad$ for $x \neq 0$

You are given that $\quad \int_{-5}^{5} f(x)\ dx = 2 \qquad \int_{-5}^{5} |f(x)|\ dx = 16$

Find $\quad \int_{-5}^{5} f(|x|)\ dx = -2B$



A, B areas

$A - B = 2$

$A + B = 16$

$A = 9,\ B = 7$

A $\quad -14$ $\qquad$ B $\quad -4$ $\qquad$ C $\quad 0$ $\qquad$ D $\quad 4$ $\qquad$ E $\quad 14$

10.

The four digit number 4284 is such that any two consecutive digits from it make a multiple of 14.
Another number N has this same property, but it has 50 digits and the first digit is 9.

What is the last digit of N? 14 28 42 56 70 84 98

A 1 B 2 C 4 D 8 E 9

9 8 4 2 8 4 2 . . . 8

48 digits

11.

Find the minimum value of the function $2^{(2x+1)} - 2^{(x+3)} - 2$

A -10 B -8 C $-\frac{25}{8}$ D -2 E $-\frac{25}{16}$

Let $a = 2^x$

$a^2 = 2^{2x}$

$2a^2 - 8a - 2$

$2(a^2 - 4a - 1)$

$2[(a-2)^2 - 5]$

$2(a-2)^2 - 10$

12.

The line $y = 2$ divides the circle $x^2 + y^2 = 8$ into two segments.

What is the area of the smaller segment?

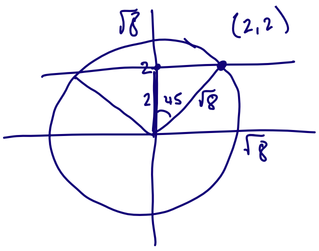
A $2\pi - 4$

B π

C 2π

D $\frac{\sqrt{2}}{2}\pi - 1$

E $16\pi - 32$



Area Sector $= \frac{1}{4}\pi \times 8 = 2\pi$

Area $\triangledown = \frac{1}{2} \times 8 = 4$

Segment : $2\pi - 4$

13.

Find the real non-zero solution to the equation $\dfrac{9^{(4^x)}}{27^{(2^x)}} = \dfrac{1}{3}$.

A $\log_3 2$ B $\log_2 3$ C -1 D $-\log_2 3$ E $-\log_3 2$

$(3^2)^{4^x} \times 3 = (3^3)^{2^x}$

$2 \times 4^x + 1 = 3 \times 2^x$

Let $a = 2^x$

$4^x = (2^2)^x = a^2$

$2a^2 + 1 = 3a$

$2a^2 - 3a + 1 = 0$

$(2a-1)(a-1) = 0$

$a = \frac{1}{2} \quad a = 1$

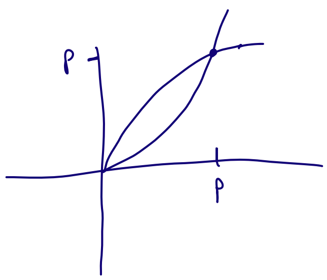
$x = -1 \quad x = 0$

14.

Find the area between the curves with equations $y = \sqrt{px}$ and $x = \sqrt{py}$

where p is a positive constant

A $\frac{1}{3}p^3$ B $\frac{2}{3}p^2 - \frac{1}{2}p^3$ C $\frac{1}{3}p^2$ D $\frac{2}{3}p^{\frac{3}{4}} - \frac{1}{3}p^2$ E p^3



$y = \sqrt{p}\, x^{1/2}$

$x = \sqrt{p}\, y^{1/2}$

$y = \frac{x^2}{p}$

$\sqrt{p}\, x^{1/2} = \frac{x^2}{p}$

$x^{3/2} = p^{3/2}$

$x = p$

$A = \int_0^p \sqrt{p}\, x^{1/2} - \frac{1}{p}x^2 \, dx$

$= \left[\frac{2}{3}\sqrt{p}\, x^{3/2} - \frac{1}{3p}x^3 \right]_0^p$

$= \frac{2}{3}p^2 - \frac{1}{3p}p^3 = \frac{1}{3}p^2$

15.

The function f is such that for every integer n $\displaystyle\int_0^n f(x)\ dx = \frac{1}{2}n(n+1)$

Evaluate $\displaystyle\sum_{r=1}^{5}\left(\int_r^{r+2} f(x)\ dx\right)$

$n=1 \quad 1 \qquad n=2 \quad 3$

$n=6 \quad 21$

$n=7 \quad 28$

A 7 B 14 C 15 D 28 E 45

$\sum_{r=1}^{5} = \int_1^3 f(x)\,dx + \int_2^4 + \int_3^5 + \int_4^6 + \int_5^7$

$= \int_1^7 f(x)\,dx + \int_2^6 f(x)\,dx$

$= \int_0^7 f(x)\,dx - \int_0^1 f(x)\,dx + \int_0^6 f(x)\,dx - \int_0^2 f(x)\,dx$

$= 28 - 1 + 21 - 3$

$= 45$

16.

Given that $f(x) = log\left(\frac{1+x}{1-x}\right)$, where $-1 < x < 1$

then $f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right)$ is equal to

A $-f(x)$

B $f(x)$

C $3f(x)$

D $(f(x))^2$

E $(f(x))^3$

$$= \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right) - \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right) - \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left(\frac{(1+x)^3}{(1-x)^3}\right) - \log\left(\frac{(1+x)^2}{(1-x)^2}\right) = \log\left(\frac{(1+x)}{(1-x)}\right) = f(x)$$

17.

The minimum value of the function $x^4 - (px)^2$ is -16 where p is a real number.

Find the minimum value of the function $x^2 + px + 5$

A -5 B $5-\sqrt{2}$ C $2\sqrt{2}$ D 3 E 5

$$x^4 - p^2x^2 = \left(x^2 - \tfrac{1}{2}p^2\right)^2 - \tfrac{1}{4}p^4$$

$$-\tfrac{1}{4}p^4 = -16$$

$$p^4 = 64$$

$$p^2 = 8$$

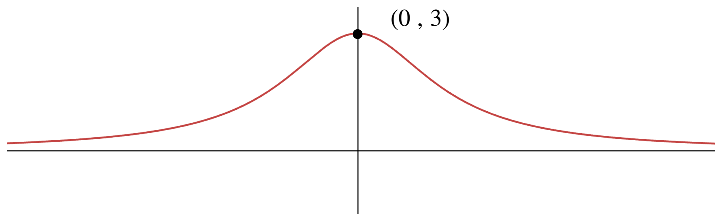
$$x^2 + px + 5 = \left(x + \tfrac{1}{2}p\right)^2 + 5 - \tfrac{1}{4}p^2$$

$$\text{Min} = 5 - \tfrac{1}{4}p^2 = 5 - 2 = 3$$

18.

The diagram shows the graph of $y = f(x)$.

The function f attains its maximum value at (0,3) and $f(x) > 0$ for all values of x.

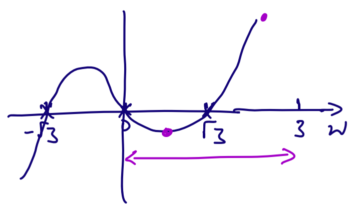


Let $w = f(x)$

$0 < w \leq 3$

$w^3 - 3w$

$w(w^2 - 3)$



Find the difference between the maximum and minimum values of $(f(x))^3 - 3f(x)$

A 0 B 4 C 12 D 18 E 20

Min at $3w^2 - 3 = 0$

$w = 1$

Max at $w = 3$

$w = 1: \ 1 - 3 = -2$

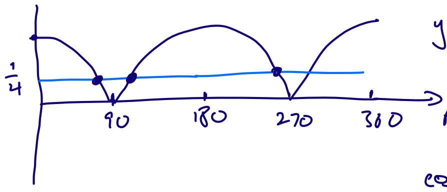
$w = 3: \ 27 - 9 = 18$

Diff $= 20$

19.

The equation $\quad cos^2(4^{sin\theta} \times 30^\circ) = \dfrac{1}{4} \quad$ has exactly two solutions in the range $\;0^\circ \le \theta \le x^\circ$

What is the range of all possible values of x ?



$y = \cos^2 A = \frac{1}{4}$

A $\quad 30 \le x < 150$

B $\quad 60 \le x < 180$

C $\quad 60 \le x < 390$

D $\quad 90 \le x < 150$

E $\quad 90 \le x < 390$

$\cos A = \pm\frac{1}{2}$

$A = 60, 120, 240$

$\cos(4^{\sin\theta} \times 30) = -\frac{1}{2}$

$4^{\sin\theta} \times 30 = 120,$

$4^{\sin\theta} = 4$

$\sin\theta = 1$

$\theta = 90$

$\cos(4^{\sin\theta} \times 30) = \frac{1}{2}$

$4^{\sin\theta} \times 30 = 60$

$4^{\sin\theta} = 2$

$\sin\theta = \frac{1}{2}$

$\theta = 30, 150$

$4^{\sin\theta} = 2 \quad 4 \;\Big|\; \cancel{8}$

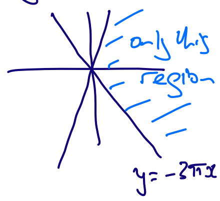
$\sin\theta = \frac{1}{2} \quad 1$

$\theta = 30, 90, 150$

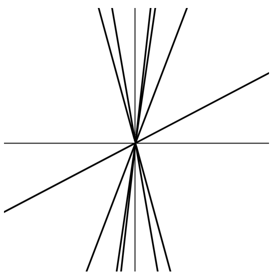
20.

Which of the following sketches shows the graph of $\quad sin\left(\dfrac{y}{x}\right) = \dfrac{1}{2} \quad$ for $\quad -3\pi x \le y \le 3\pi x$

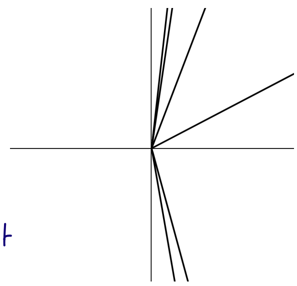
$y \le 3\pi x$



A



B



$\sin\left(\frac{y}{x}\right) = \frac{1}{2}$

$\frac{y}{x} = \frac{\pi}{6}, \frac{5\pi}{6} \quad \frac{13\pi}{6} \quad \frac{17\pi}{6}$

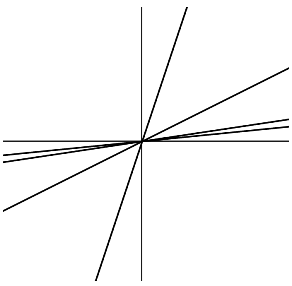
4 lines +ve gradient

$-\frac{7\pi}{6} - \frac{11\pi}{6}$

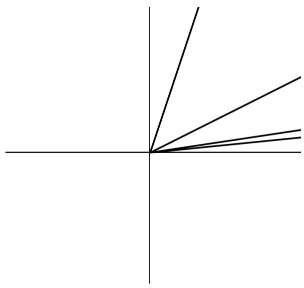
2 lines -ve gradient

$x > 0$ only

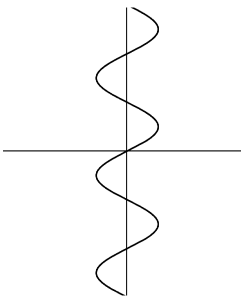
C



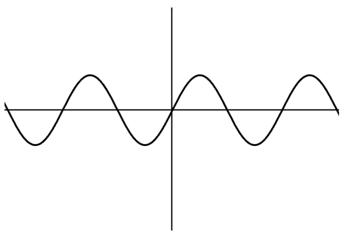
D



E



F



Mock TMUA Set C: Paper 2

20 questions
75 minutes
No calculator allowed

1.

Given that $\dfrac{dy}{dx} = 6x^2 - \dfrac{4x-3}{x^4}$, $\quad x \neq 0 \quad$ and $y = 6$ when $x = 1$, find y in terms of x.

A $\quad y = 12x + 12x^{-2} - 12x^{-3} - 6$

B $\quad y = 12x + 12x^{-4} - 12x^{-5} - 6$

C $\quad y = 2x^3 + x^{-4} - x^{-3} + 4$

D $\quad y = 2x^3 + 2x^{-2} - x^{-3} + 3$

E $\quad y = 2x^3 + 2x^{-2} + x^{-3} + 1$

$\frac{dy}{dx} = 6x^2 - 4x^{-3} + 3x^{-4}$

$y = 2x^3 + 2x^{-2} - x^{-3} + c$

$6 = 2 + 2 - 1 + c \qquad c = 3$

2.

Find the complete set of values of the real constant p for which the expression

$$x^2 - 2x + px - 2 + p$$

is positive for all real values of x .

A $\quad 4 - 2\sqrt{5} < p < 4 + 2\sqrt{5}$

B $\quad p < 4 - 2\sqrt{5}$ or $p > 4 + 2\sqrt{5}$

C $\quad -2 < p < 2$

D $\quad p < -2$ or $p > 2$

E $\quad 2 < p < 6$

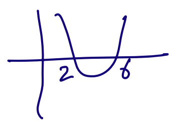
F $\quad p < 2$ or $p > 6$

$\Delta < 0$

$(p-2)^2 - 4(p-2) < 0$

$(p-2)(p-2-4) < 0$

$(p-2)(p-6) < 0$



3.

The real numbers a, b, and c are non-zero and $a \leq b$.

Which of the following statements are necessarily true?

I $\quad \dfrac{1}{a} \geq \dfrac{1}{b}$ no $a = -2$ $b = 1$

II $\quad 2^a \leq 2^b$ ✓ 2^x increasing function

III $\quad ac \leq bc$ no $c = -1$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

4.

A bag only contains $2n$ blue balls and n red balls. All the balls are identical except in colour. One ball is randomly selected and not replaced. A second ball is then randomly selected. What is the probability that at least one of the selected balls is red?

A $\quad \dfrac{n(n-1)}{3(3n-1)}$

B $\quad \dfrac{3n-1}{3(3n-1)}$

C $\quad \dfrac{4n-2}{3(3n-1)}$

D $\quad \dfrac{2n(n-1)}{3(3n-1)}$

E $\quad \dfrac{5n-1}{3(3n-1)}$

$P(\text{no red}) = \dfrac{2n}{3n} \times \dfrac{2n-1}{3n-1}$

$P(\text{at least one red}) = 1 - \dfrac{2n(2n-1)}{3n(3n-1)}$

$= \dfrac{9n^2 - 3n - 4n^2 + 2n}{3n(3n-1)}$

$= \dfrac{5n^2 - n}{3n(3n-1)}$

$= \dfrac{5n-1}{3(3n-1)}$

5.

A student attempts to prove the following statement.

> Consider the integers a and b, where a has remainder 1 when divided by 3, and b has remainder 2 when divided by 3.
>
> Then $a + b$ is always divisible by 3.

Consider the following attempt:

Let	$a = 3n + 1$ and $b = 3n + 2$	(I)
then	$a + b = 3n + 1 + 3n + 2$	(II)
so	$a + b = 6n + 3$	(III)
so	$a + b = 3(2n + 1)$	(IV)
therefore	$a + b$ is always divisible by 3.	(V)

should be $b = 3m+2$

$a+b = 3n+1+3m+2$

$= 3(n+m+1)$

Which of the following best describes this proof?

A The statement is true and the proof is completely correct.

B The statement is true but there is an error in the proof in line (I)

C The statement is true but there is an error in the proof in line (II)

D The statement is not true and there is an error in the proof in line (III)

E The statement is not true and there is an error in the proof in line (IV)

F The statement is not true and there is an error in the proof in line (V)

6.

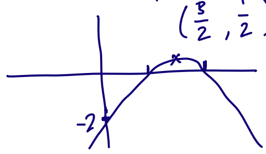
This question uses radians.

Find the number of distinct values of x that satisfy the equation

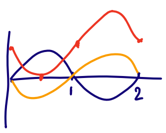
$$2(2 - x)(x - 1) = 1 - sin\pi x$$

$2(2-x)(x-1)$ -ve quadratic

$\left(\frac{3}{2}, \frac{1}{2}\right)$

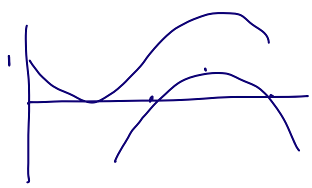


A 0 B 1 C 2 D 3 E 4



$1 - \sin\pi x$

$-\sin\pi x$



7. $1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36$

Consider the statement:

(*) Every prime number n can be written as the sum of 2 square numbers.

How many counterexamples to (*) are there in the range $0 < n < 40$

2 3 5 7 11 13 17

$1\,1$ $1\,4$ $4\,9$ $1\,16$

19 23 29 31 37

$4\,25$ $1\,36$

A 2

B 3

C 4

D 5

E 6

8.

The notation $\lfloor x \rfloor$ means the greatest integer less than or equal to x. $\frac{\pi}{2} \approx 1.57 \quad \pi \approx 3.14$

For example $\lfloor 0.8 \rfloor = 0 \qquad \lfloor 2 \rfloor = 2 \qquad \lfloor \sqrt{12} \rfloor = 3$

Evaluate the integral $\displaystyle\int_{\frac{\pi}{2}}^{\pi} x \lfloor x \rfloor \; dx = \int_{\frac{\pi}{2}}^{2} x \, dx + \int_2^3 2x \, dx + \int_3^{\pi} 3x \, dx$

A $\dfrac{26}{3}$ B $\dfrac{11\pi^2}{8} - \dfrac{13}{2}$ C $\dfrac{11\pi^2}{8}$ D $\dfrac{7\pi^3}{24}$ E $\dfrac{7\pi^3}{24} - \dfrac{13}{8}$

$= \left[\frac{1}{2}x^2\right]_{\frac{\pi}{2}}^{2} + \left[x^2\right]_2^3 + \left[\frac{3}{2}x^2\right]_3^{\pi}$

$= 2 - \frac{\pi^2}{8} + 5 + \frac{3}{2}\pi^2 - \frac{27}{2} = \frac{11\pi^2}{8} - \frac{13}{2}$

9.

A locked box has two levers, A and B, which can be positioned either left or right at any particular time. It is known that if lever A is left or lever B is right, then the box is unlocked.

Which of the following statements must be true?

A left or B right $\Rightarrow$ unlocked

A If the box is unlocked then lever A is left or lever B is right

B If the box is locked then lever A is left and lever B is right

Contrapositive

locked $\Rightarrow$ A right and B left

C If the box is unlocked then lever A is left and lever B is right

D If the box is locked then lever A is right or lever B is left

E If the box is unlocked then lever A is right or lever B is left

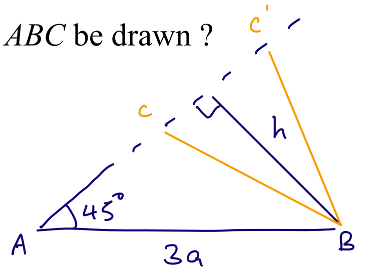
F If the box is locked then lever A is right and lever B is left

10.

A triangle ABC is to be drawn with the following measurements.

$AB = 3a$ $\quad BC = b$ $\quad$ angle $BAC = 45°$ $\quad$ where a and b are constants

For which values of b can two distinct triangles ABC be drawn ?



$\sin 45 = \dfrac{h}{3a}$

$h = \dfrac{3}{2}\sqrt{2}\, a$

$b > \dfrac{3}{2}\sqrt{2}\, a$

$b < 3a$

A $\quad b < 3a$

B $\quad a < b < 3a$

C $\quad \dfrac{3}{2}\sqrt{2}\, a < b < 3a$

D $\quad b > \sqrt{2}\, a$

E $\quad b > \dfrac{3}{2}\sqrt{2}\, a$

11.

P $\quad$ Q

Consider the statement: $\quad f(x) > g(x) \quad$ for all real values of $x > a$

Which one of the following is a negation of this statement?

not P for at least one Q

A $\quad f(x) \le g(x)$ $\quad$ for at least one real value of $\quad x > a$

B $\quad f(x) \le g(x)$ $\quad$ for all real values of $\quad x > a$

C $\quad f(x) \le g(x)$ $\quad$ for at least one real value of $\quad x \le a$

D $\quad f(x) \le g(x)$ $\quad$ for no real values of $\quad x > a$

E $\quad f(x) > g(x)$ $\quad$ for at least one real value of $\quad x \le a$

F $\quad f(x) > g(x)$ $\quad$ for at least one real value of $\quad x > a$

G $\quad f(x) > g(x)$ $\quad$ for all real values of $\quad x \le a$

H $\quad f(x) > g(x)$ $\quad$ for no real values of $\quad x > a$

12.

The function $F(n)$ is defined for all positive integers as follows:

$F(1) = 1$ and for all $n \geq 2$

$F(n) = F(n-1) + 5$	if 5 divides n but 2 does not divide n	5
$F(n) = F(n-1) + 2$	if 2 divides n but 5 does not divide n	2 4 6 8
$F(n) = F(n-1) - 1$	if 2 and 5 both divide n	10
$F(n) = F(n-1)$	if neither 2 nor 5 divides n	3 7 9

The value of $F(301)$ is equal to

A 300 B 301 C 361 D 363 E 372

n =	1	2	3	4	5	6	7	8	9	10	...	300	301
$F(n)$:	1	3	3	5	10	12	12	14	14	13		361	361

every group of 10 adds 12

30 groups
$30 \times 12 = 360$

13.

Consider the following statements for real values of x.

P: $\sqrt{x+12} \geq x$

Q: $-3 \leq x \leq 4$

$x + 12 = x^2$
$x^2 - x - 12 = 0$
$(x-4)(x+3) = 0$
$x = -3, 4$

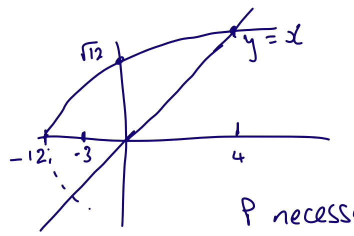
Which one of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **sufficient** but **not necessary** for Q

D P is **necessary** but **not sufficient** for Q



P necessary for Q
(If Q then P)

But if $x = -6$ then P is true
$\sqrt{6} \geq -6$
but Q not true

14.

A list consists of k integers, and the mean of these is calculated to be m . Total $= km$

When an integer a is added to this list, the mean decreases by 1. $\frac{km+a}{k+1} = m-1$

When a further integer b is added to the new list, the mean decreases again by another 1.

Which one of the following statements is true? $\frac{km+a+b}{k+2} = m-2$

A $\quad m = k + 4$

B $\quad k < m$

C $\quad a + b = 2(m - k)$

D $\quad a(m - 1) = b(m - 2)$

E $\quad a - b = 2$

$km + a = mk - k + m - 1$
$a = m - k - 1$

$km + a + b = mk - 2k + 2m - 4$
$a + b = 2m - 2k - 4$
$b = m - k - 3$

$a - b = 2$

15.

Consider the following statement:

If $f'(x) > 0$ *for all real* x *then* $f(x+1) > f(x)$ *for all real* x

need positive gradient but $f(x+1) \leq f(x)$ for some x

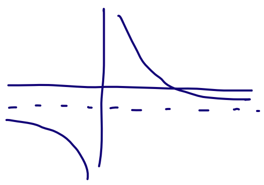
Which function provides a counterexample:

A $\quad f(x) = 4^x$ $\qquad$ B $\quad f(x) = 4x^2 + 1$ $\qquad$ C $\quad f(x) = 4x^3$

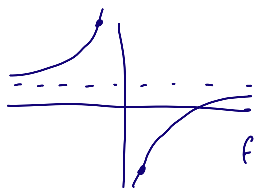
D $\quad f(x) = \frac{4 - x}{x}$ $\qquad$ E $\quad f(x) = \frac{x - 1}{4x}$

D: $\frac{4}{x} - 1$

$f'(x) < 0$



E: $\frac{1}{4} - \frac{1}{4x}$



positive gradient

$f(\frac{1}{2}) < f(-\frac{1}{2})$

16.

Given that $(x^2 + x + 2)^{24} = a_0 + a_1x + \ldots + a_{48}x^{48}$

Find the value of $\quad a_0 + a_2 + a_4 + \ldots + a_{48} = k$

$x = 1 \quad 4^{24} = a_0 + a_1 + \ldots + a_{48}$

$x = -1 \quad 2^{24} = a_0 - a_1 + a_2 - \ldots - a_{47} + a_{48}$

$2^{48} + 2^{24} = 2k$

$k = 2^{47} + 2^{23} = 2^{23}(2^{24} + 1)$

A $\quad 2^{24}(2^{22} + 1)$

B $\quad 2^{24}(2^{22} - 1)$

C $\quad 2^{23}(2^{24} - 1)$

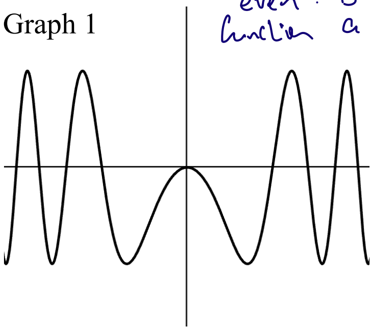
D $\quad 2^{23}(2^{24} + 1)$

E $\quad 2^{23}(2^{25} + 1)$

17.

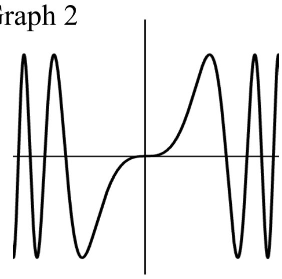
Graphs 1 and 2 represent functions of the form $y = a\sin(x^b)$ where a, b are non-zero integers

Graph 1



even function : b even, positive
a negative

Graph 2



odd function

b odd, positive
a positive

Which of the following statements are necessarily true?

I In Graph 1, $a < 0$ and b is even ✓

II In Graph 2, $a > 0$ and $b < 0$ no : b is positive

III $b > 0$ in both Graph 1 and Graph 2 ✓ negative b would give asymptote at $x = 0$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

18.

$P(x)$ and $Q(x)$ are defined as follows: $P(x) = 3^{(2x-1)} - 3^{(x+1)} + 3$ $Q(x) = 3^x - 3$.

What is the largest value of x such that $P(x)$ and $Q(x)$ are in the ratio 5 : 3 respectively.

A $log_3 2$ B $log_3 5$ C $log_3 12$ D 3 E 5

Let $a = 3^x$

$P: \frac{1}{3}a^2 - 3a + 3$

$Q: a - 3$

$\frac{P}{Q} = \frac{5}{3}$

$3P = 5Q$

$a^2 - 9a + 9 = 5a - 15$

$a^2 - 14a + 24 = 0$

$(a-2)(a-12) = 0$

$a = 2, 12$

$3^x = 12$

$x = \log_3 12$

19.

A curve has the equation $y = ax^3 + bx^2 + c$

The curve has a maximum stationary point at $x = 0$ and a minimum stationary point in the 4th quadrant (the region where $x > 0$ and $y < 0$).

Which of the following set of conditions is **sufficient** to ensure this?

A $\quad \cancel{a < 0}, b < 0, c < 0$

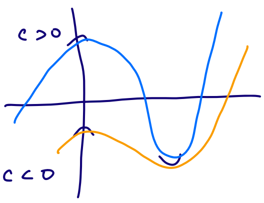
B $\quad a > 0, b < 0, c < 0$

C $\quad \cancel{a < 0}, b > 0, c < 0$

D $\quad a > 0, b < 0, c > 0$

E $\quad a > 0, \cancel{b > 0}, c > 0$

F $\quad$ None of the above



need $\underline{a > 0}$

for 4th quadrant

$y' = 3ax^2 + 2bx$
$= x(3ax + 2b)$

$x = 0 \quad x = -\frac{2b}{3a} > 0$

$\frac{2b}{3a} < 0$

$b < \frac{3}{2}a$

$y'' = 6ax + 2b$

Max at $x = 0$ $\quad 2b < 0$
$b < 0$

Min at $x = -\frac{2b}{3a}$ $\quad -4b + 2b > 0$
$2b < 0$

Need $\underline{b < 0}$

At $x = -\frac{2b}{3a}$ $\quad y = a\left(\frac{-8b^3}{27a^3}\right) + b\left(\frac{4b^2}{9a^2}\right) + c$

$y < 0$ for 4th quadrant

$-\frac{8b^3}{27a^2} + \frac{4b^3}{9a^2} + c < 0$

$\frac{4b^3}{27a^2} + c < 0$

$c < -\underbrace{\left(\frac{4b^2}{27a^2}\right)b}_{+ve}$

sufficient if $c < 0$

20.

f is a function and a *is* a real number.
Given that exactly one of the following statements is true, which one is it?

P : $a > 0$
Q : $f(a) > 0$

A $\quad a \leq 0$ **only if** $f(a) \leq 0$ $\quad$ If P' then Q' : If Q then P

B $\quad f(a) > 0$ **if** $a > 0$ $\quad$ If P then Q

C $\quad f(a) > 0$ is **sufficient** for $a > 0$ $\quad$ If Q then P

D $\quad f(a) \leq 0$ is **necessary** for $a \leq 0$ $\quad$ If P' then Q' $\quad$ If Q then P

E $\quad$ **If** $f(a) > 0$ **then** $a > 0$ $\quad$ If Q then P

F $\quad a > 0$ **if** $f(a) > 0$ $\quad$ If Q then P