1) The line $y = mx + 4$ where $m > 0$ is the normal to the curve $y = 6 - x^2$ at the point (p,q). What is the value of p?

A $\frac{\sqrt{2}}{6}$ B $-\frac{\sqrt{2}}{6}$ C $\sqrt{\frac{3}{2}}$ D $\pm\sqrt{\frac{3}{2}}$ E $\sqrt{\frac{5}{2}}$

$\frac{dy}{dx} = -2x$ At (p,q) $\frac{dy}{dx} = -2p$

Grad of normal $= \frac{1}{2p} = m$

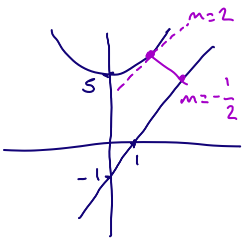
$q = mp + 4$
$q = 6 - p^2$
$mp + 4 = 6 - p^2$
$p^2 + mp - 2 = 0$

$p^2 = 2 - \frac{1}{2} = \frac{3}{2}$
$p = \pm\sqrt{\frac{3}{2}}$

$p > 0$ (as $m > 0$) $p = \sqrt{\frac{3}{2}}$

2) Find the shortest distance between the line $y = 2x - 1$ and the curve $y = x^2 + 5$

A 2 B $\sqrt{5}$ C $\sqrt{\frac{5}{2}}$ D 3 E 5



don't intersect

$m = -\frac{1}{2}$
$(1,6)$

$\frac{dy}{dx} = 2x = 2$
$x = 1$ $(1,6)$
$y - 6 = -\frac{1}{2}(x - 1)$
$y = -\frac{x}{2} + \frac{13}{2}$

$2x - 1 = -\frac{x}{2} + \frac{13}{2}$
$\frac{5x}{2} = \frac{15}{2}$
$x = 3$
$(3,5)$

$(1,6)$ $(3,5)$
$\sqrt{(6-5)^2 + (1-3)^2}$
$= \sqrt{5}$

3) A line is drawn normal to the curve $y = \frac{2}{x}$ at the point where $x = 1$.

This line cuts the x-axis at P and y-axis at Q. Find the length of PQ.

A $\frac{3}{2}$ B $\frac{3}{2}\sqrt{5}$ C $\sqrt{\frac{15}{2}}$ D $2\sqrt{5}$ E 3

$y = \frac{2}{x} = 2x^{-1}$ $\frac{dy}{dx} = -\frac{2}{x^2}$

At $x = 1$, $y = 2$ $\frac{dy}{dx} = -2$

grad $PQ = \frac{1}{2}$

$y - 2 = \frac{1}{2}(x - 1)$
$y = \frac{x}{2} + \frac{3}{2}$
$P\ (-3, 0)$
$Q\ (0, \frac{3}{2})$

$|PQ| = \sqrt{9 + 9/4}$
$= 3\sqrt{1 + 1/4}$
$= \frac{3}{2}\sqrt{5}$

4) The line $y = mx + 2$ passes through the points $(5\,, log_3 p)$ and $(log_3 p\,, 2)$

What is the difference between the possible values of p?

(A) 8 B 3 C $\frac{2}{5}$ D 2 E 10

$\log_3 p = 5m + 2$ $\quad$ $2 = m\log_3 p + 2$

$2 = m(5m+2)+2$ $\quad$ $\log_3 p = 2$ $\quad$ $\log_3 p = 0$

$m=0 \quad m = -\frac{2}{5}$ $\quad$ $p = 9$ $\quad$ $p = 1$ $\quad$ $9 - 1 = 8$

5) The line segment joining the points (2,2) and (6,8) is a diameter of a circle.

This circle is translated by 3 units in the positive x-direction, then reflected in the x-axis, and then enlarged by a scale factor of 2 about the centre of the resulting circle.

Find the equation of the final circle.

A $(x-7)^2 + (y-5)^2 = 26$ B $(x-7)^2 + (y+5)^2 = 26$

C $(x-1)^2 + (y-5)^2 = 52$ D $(x-1)^2 + (y+5)^2 = 52$

(E) $(x-7)^2 + (y+5)^2 = 52$ F $(x-1)^2 + (y-5)^2 = 26$

Midpoint $= (4,5)$ $\quad r = \sqrt{2^2+3^2} = \sqrt{13}$

$(x-4)^2 + (y-5)^2 = 13$

3 units +ve x-direction $\quad (x-7)^2 + (y-5)^2 = 13$

reflected in x-axis $\quad (x-7)^2 + (y+5)^2 = 13$

enlarged S.F. 2 $\quad (x-7)^2 + (y+5)^2 = 52$

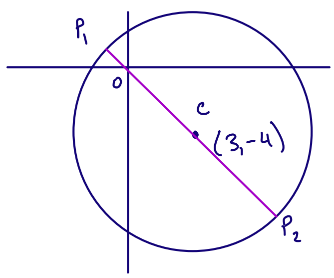
6) A point P lies on the curve with equation $x^2 + y^2 - 6x + 8y = 24$

What is the difference between the greatest and least possible values of the length OP, where O is the origin.

A 2 B 7 (C) 10 D 12 E 14

$(x-3)^2 - 9 + (y+4)^2 - 16 = 24$ $\quad$ Centre $(3,-4)$ $\quad r = 7$

$(x-3)^2 + (y+4)^2 = 49$



$CP_1 = CP_2 = 7$

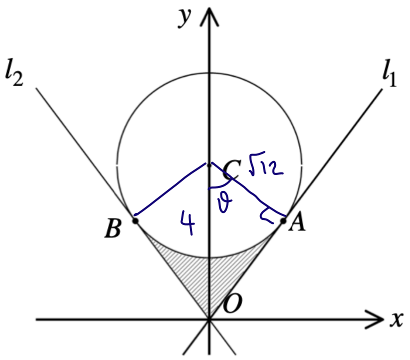
$OC = \sqrt{3^2+4^2} = 5$

$OP_1 = 7 - 5 = 2$ $\quad OP_2 = 5 + 7 = 12$

Difference $= 12 - 2 = 10$

7) The diagram shows a circle with equation $x^2 + (y-4)^2 = 12$ and lines l_1 and l_2 which are tangents to the circle at A and B.

Find the area of the shaded region enclosed by the circle and the lines l_1 and l_2.



Circle centre $(0,2)$ $r = \sqrt{12}$

$OA = \sqrt{16-12}$
$= 2$

$\cos\theta = \frac{\sqrt{12}}{4} = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$

Area $\triangle COA = \frac{1}{2} \times 2 \times \sqrt{12}$
$= \sqrt{12}$

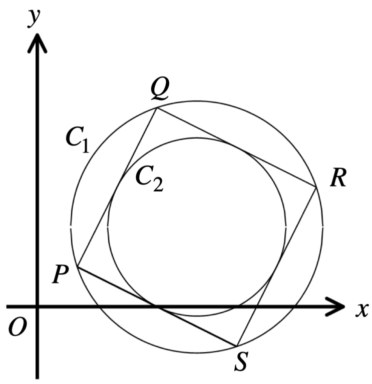
Area sector $= \frac{1}{2} \times 12 \times \frac{\pi}{6}$
$= \pi$

A $\pi - 2$ B $2\sqrt{3} - \pi$ C $\frac{3\pi}{2}$ D $4\sqrt{3} - 2\pi$ E $2\sqrt{3} + \pi$

Shaded Area $= 2\left(\sqrt{12} - \pi\right) = 4\sqrt{3} - 2\pi$

8) The diagram shows a square PQRS with vertices at the points P (1,1), Q (3,5), R (7,3) S (5,-1).
The square is circumscribed by the circle C_1 and inscribed by the circle C_2

Find the area of the annulus between these two circles.



Midpoint PR $= (4,2) =$ centre

radius $C_1 = \sqrt{1^2+3^2} = \sqrt{10}$
(centre to Q)

radius $C_2 = \sqrt{1^2+2^2} = \sqrt{5}$
(centre to mid QR (5,4))

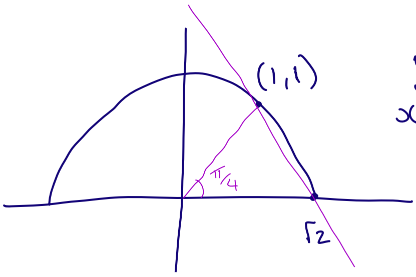
Area $= 10\pi - 5\pi = 5\pi$

A $(\sqrt{10} - \sqrt{5})\pi$ B 2π C 5π D $\frac{5\pi}{2}$ E $\sqrt{5}\pi$

9) Find the area bounded by the graphs $y = \sqrt{2 - x^2}$ and $x + (\sqrt{2} - 1)y = \sqrt{2}$

(1, 1) lies on both

(a) $\frac{\pi}{4} - \frac{1}{\sqrt{2}}$ (b) $\frac{sin\sqrt{2}}{\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi^2}{6}$



$y^2 = 2 - x^2$
$x^2 + y^2 = 2$

$y = \frac{-x + \sqrt{2}}{\sqrt{2} - 1}$

$(0, \frac{\sqrt{2}}{\sqrt{2}-1})$ $(\sqrt{2}, 0)$
$(1, 1)$

Sector $= \frac{1}{8} \times \pi \times 2 = \frac{\pi}{4}$

$\triangle = \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Area $= \frac{\pi}{4} - \frac{1}{\sqrt{2}}$

10) The lines given by the following equations are perpendicular.

① $(1 + \sqrt{3})y = px + 5$ ② $y = (2 - \sqrt{3})x + 8$

What is the value of p ?

Gradient ① : $\frac{p}{1+\sqrt{3}}$ ② $2-\sqrt{3}$

⊥ : $\frac{p}{1+\sqrt{3}} = \frac{-1}{2-\sqrt{3}}$

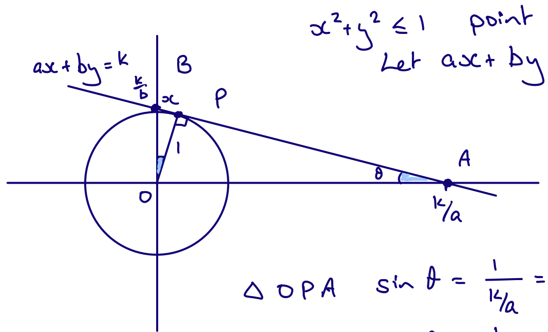
$p = \frac{1+\sqrt{3}}{\sqrt{3}-2} \times \frac{(\sqrt{3}+2)}{(\sqrt{3}+2)} = \frac{3\sqrt{3}+5}{3-4}$
$= -3\sqrt{3} - 5$

A $-5 - 3\sqrt{3}$
B $-5 + 3\sqrt{3}$
C $5 - 3\sqrt{3}$
D $5 + 3\sqrt{3}$

11) Let a and b be positive real numbers such that $a \le b$
Given that $x^2 + y^2 \le 1$ then the largest value that $a\,x + b\,y$ can equal is:

A $a + b$
B b
C $\sqrt{a^2 + b^2}$
D $a^2 + ab + b^2$
E $\frac{1}{a} + \frac{1}{b}$

$x^2 + y^2 \le 1$ point is within the unit circle
Let $ax + by = k$ where k is the largest value



$\triangle OPA$ $\sin\theta = \frac{1}{k/a} = \frac{a}{k}$

$\triangle OPB$ $\cos\theta = \frac{1}{k/b} = \frac{b}{k}$

using $\sin^2\theta + \cos^2\theta = 1$

$\frac{a^2}{k^2} + \frac{b^2}{k^2} = 1$ $k^2 = a^2 + b^2$
$k = \sqrt{a^2 + b^2}$