updated 12 Aug 2026

TMUA style questions - Modulus Function

1. x and y satisfy $\quad |3-2x| \le 7 \quad$ and $\quad |y+3| \le 6$

What is the greatest possible value of $|x+y|$

A 7

B 8

C 11

D 14

E There is no greatest possible value

* equivalently $-3+2x \le 7$

$3-2x \le 7$ or * $3-2x \ge -7$

$x \ge -2$ $\quad x \le 5$ $\quad -2 \le x \le 5$

$y+3 \le 6$ or $y+3 \ge -6$

$y \le 3$ $\quad y \ge -9$ $\quad -9 \le y \le 3$

max $|x+y| = |-2+-9| = 11$

2. Find the set of values of x for which $\quad |x^2-4| < 4-2x$

A $-4 < x < -2$

B $-4 < x < 0$

C $-4 < x < 2$

D $-2 < x < 0$

E $0 < x < 2$

$2x-4 < x^2-4 < 4-2x$

$x^2-4 \ge 0 \quad x \ge 2, x \le -2$

$x^2-4 < 4-2x$

$x^2+2x-8 < 0$

$(x+4)(x-2) < 0$

$-4 < x < 2$

$\Rightarrow \underline{-4 < x \le -2}$

$x^2-4 < 0 \quad -2 < x < 2$

$x^2-4 > 2x-4$

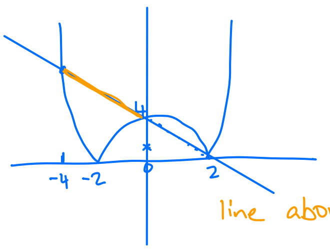
$x^2-2x > 0$

$x(x-2) > 0$

$x < 0, \ x > 2$

$\underline{-2 < x < 0}$

(can solve this one graphically or algebraically



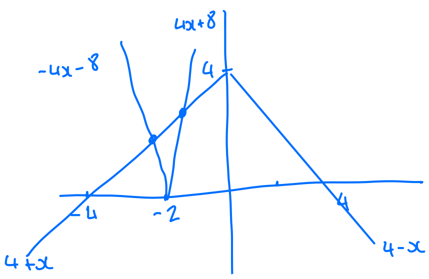
$\boxed{-4 < x < 0}$

line above quadratic

3. What is the product of the solutions of the following equation

$$4 - |x| = |4x+8|$$

A $\frac{16}{15}$ $\quad$ B $\frac{48}{25}$ $\quad$ C $\frac{16}{5}$ $\quad$ D 4 $\quad$ E $\frac{16}{3}$



$4x+8 = 4+x$

$x = -\frac{4}{3}$

$-4x-8 = 4+x$

$5x = -12$

$x = -\frac{12}{5}$

Product $= \frac{48}{15} = \frac{16}{5}$

4. How many solutions are there to the following equation:

$$|x^3| + |x-2| = |4x|$$

A 0

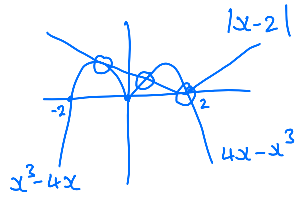
B 1

C 2

D 3

E 4

$|x-2| = |4x| - |x^3|$



$x \geq 0$ $\quad |x-2| = 4x - x^3$ $\quad$ always 2 solutions (see graph)

$x < 0$ $\quad |x-2| = x^3 - 4x$ $\quad$ 0/1/2 solutions?

$2-x = x^3-4x$

$x^3 - 3x - 2 = 0$

$(x+1)(x^2-x-2) = 0$

$(x+1)(x-2)(x+1) = 0$

repeated root at $x=-1$ $\quad \therefore$ 1 extra solution $\quad$ Total 3 solutions

5. What is the minimum value of the function:

$$|x+1| + |x-1| + |x-2| + |x-4|$$

A 0

B 2

C 4

D 6

E 8

Need shortest distance from $-1, 1, 2, 4$

These points are evenly spaced about midpoint $\frac{3}{2}$

When $x = \frac{3}{2}$ $\quad 2.5 + 0.5 + 0.5 + 2.5 = 6$

6. Find the area between the curves with equations $y = 2|x+1|$ and $y = 6 - 2|x+1|$

A $\sqrt{5}$

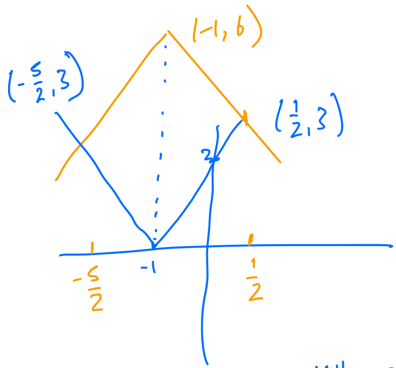
B 6

C $3\sqrt{5}$

D 9

E $6\sqrt{5}$

F 18



$2(x+1) = 6 - 2(x+1)$

$4x + 4 = 6$

$x = \frac{1}{2}$ $\quad y = 3$

$-2x - 2 = 6 + 2x + 2$

$4x = -10$

$x = -\frac{5}{2}$ $\quad y = 3$

Kite shape

Area $= \frac{1}{2} \times 6 \times 3 = 9$

7. x and y satisfy $|y - x| \le 2$ and $y \le 4 - x^2$

What is the greatest possible value of $|xy|$

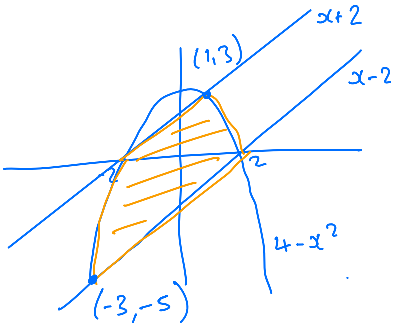
A 3 B 4 C 8 D 15 E 16

$-2 \le y - x \le 2$

$y \ge x - 2$

$y \le x + 2$

x, y lie in



$x + 2 = 4 - x^2$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$(1,3)$ $(-2,0)$

$x - 2 = 4 - x^2$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

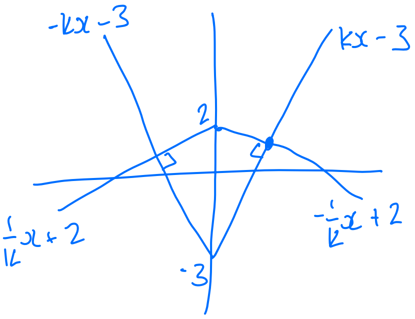
$(2,0)$ $(-3,-5)$

Max $|xy| = |-3 \times -5| = 15$

8. Consider the finite area between the curves $y = k|x| - 3$ and $y = -\dfrac{1}{k}|x| + 2$

What is the sum of the possible values of k that give an area of 10 square units?

A 2 B $\dfrac{5}{2}$ C 3 D $\dfrac{10}{3}$ E 5



$kx - 3 = -\frac{1}{k}x + 2$

$k^2x + x = 5k$

$A = 10$ $x = 2$ (from area of triangle $= 5 = \frac{1}{2} \times 5 \times x$)

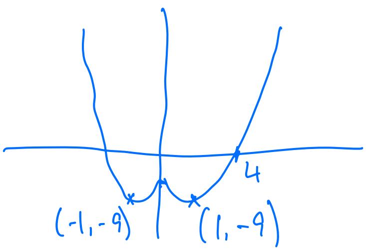
$2k^2 - 5k + 2 = 0$

$(2k-1)(k-2) = 0$

$k = \frac{1}{2}$ $k = 2$ Sum $= \frac{5}{2}$

9. Find the finite area between the coordinate axes and the curve $y = x^2 - 2|x| - 8$?

A $-\dfrac{160}{3}$ B $-\dfrac{80}{3}$ C -4 D $-\dfrac{16}{3}$ E 0



$= 2\int_0^4 y\,dx = 2\int_0^4 x^2 - 2x - 8 = 2\left[\frac{1}{3}x^3 - x^2 - 8x\right]_0^4$

$= 2\left(\frac{64}{3} - 16 - 32\right)$

$= 2\left(\frac{64}{3} - \frac{144}{3}\right) = 2\left(-\frac{80}{3}\right) = -\frac{160}{3}$

10. Evaluate the following integral $\int_0^3 x|6-3x| \, dx$

$6-3x \geq 0$
$x \leq 2$
split integral at $x=2$

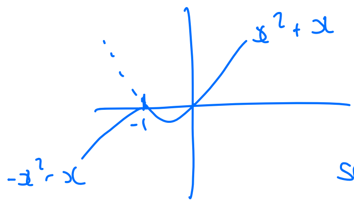
A 0 B 4 C 8 D 12 E 15

$= \int_0^2 6x - 3x^2 \, dx + \int_2^3 3x^2 - 6x \, dx$

$= \left[3x^2 - x^3\right]_0^2 + \left[x^3 - 3x^2\right]_2^3$

$= 12 - 8 + 27 - 27 - (8 - 12) = 8$

11. Evaluate $\int_{-2}^{1} x\left|(x+1)\right| \, dx$



split integral at $x=-1$

A $-\frac{3}{2}$ B $-\frac{1}{3}$ C $-\frac{1}{6}$ D $\frac{1}{6}$ E $\frac{1}{3}$ F $\frac{3}{2}$

$= \int_{-2}^{-1} -x^2 - x \, dx + \int_{-1}^{1} x^2 + x \, dx$

$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_{-2}^{-1} + \left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^{1}$

$= \frac{1}{3} - \frac{1}{2} - \left(\frac{8}{3} - 2\right) + \frac{1}{3} + \frac{1}{2} - \left(-\frac{1}{3} + \frac{1}{2}\right) = -\frac{1}{6}$

12. Evaluate the following integral $\int_0^2 2x(|1-x|) - x|x| \, dx$

$0 \leq x \leq 1$ $|x| = x$
$|1-x| = 1-x$
$1 \leq x \leq 2$ $|x| = x$
$|1-x| = x-1$

A $-\frac{2}{3}$ B $-\frac{8}{3}$ C -4 D $-\frac{16}{3}$ E 0

$= \int_0^1 2x - 3x^2 \, dx + \int_1^2 x^2 - 2x \, dx$

$= \left[x^2 - x^3\right]_0^1 + \left[\frac{1}{3}x^3 - x^2\right]_1^2$

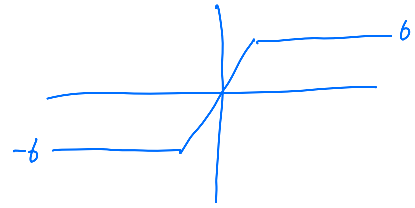
$= 0 + \frac{8}{3} - 4 - \frac{1}{3} + 1 = \frac{7}{3} - 3 = -\frac{2}{3}$

13. Arrange the following integrals in order from smallest to largest:

P: $\displaystyle\int_{-2}^{2} \left|3+x\right| - \left|3-x\right| \;\; dx$ odd function zero

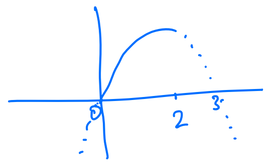
$x > 3$ $3+x-(x-3)=6$
$-3<x<3$ $3+x-(3-x)=2x$
$x<-3$ $-3-x-(3-x)=-6$

P



Q: $\displaystyle\int_{0}^{2} x\left|3-x\right| \;\; dx$ +ve

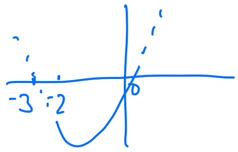
Q : $3x - x^2 = x(3-x)$ $0 \le x \le 2$



above x-axis

R: $\displaystyle\int_{-2}^{0} x\left|3+x\right| \;\; dx$ -ve

R : $3x + x^2 = x(x+3)$ $-2 \le x \le 0$



below x-axis

A P Q R

B P R Q

C Q P R

D Q R P

E R P Q

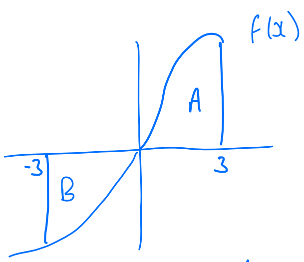
F R Q P

14. The function $f(x)$ is such that $f(0) = 0$ and $xf(x) > 0$ for $x \neq 0$

You are given that (1) $\displaystyle\int_{-3}^{3} f(x) \;\; dx = 0$ (2) $\displaystyle\int_{-3}^{3} f(|x|) \;\; dx = 6$

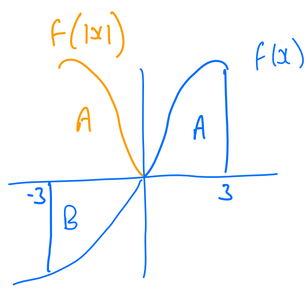
Find $\displaystyle\int_{-3}^{0} |f(x)| \;\; dx$

A -6 B -3 C 0 D 3 E 6

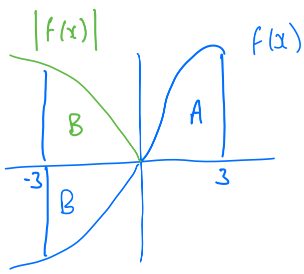


possible f(x) with areas A, B

(1) $A - B = 0$

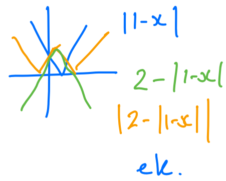


(2) $2A = 6$
$A = 3$, $B = 3$



$\displaystyle\int_{-3}^{0} |f(x)| \, dx = B$

or series of sketches



15. How many solutions are there to $\left|3-\left|2-\left|\overset{1-x}{\cancel{x-1}}\right|\right|\right|-1=0$

A 2 B 3 C 4 D 5 E 6

$3-|2-|1-x||=\pm 1$

$|2-|1-x||=2,4$

$2-|1-x|=\pm 2, \pm 4$ but can't be 4 as $|1-x|>0$

$|1-x|=4,0,6$

$|x-1|=0,4,6$

$x-1=0,\pm 4,\pm 6$

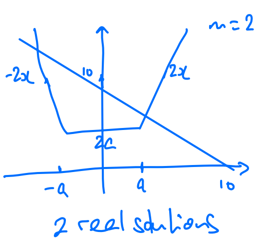
$x=1,-3,5,-5,7$

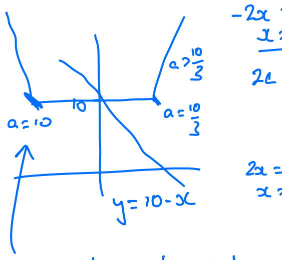
16. The equation $|x-a|+|x+a|=10-x$ where $a>0$ has exactly 1 real solution.
What is the value of a ?

A 0 B $\frac{5}{3}$ C $\frac{10}{3}$ D 5 E 10

$-a \le x < a$
$x < -a$
$x \ge a$

$-x+a+x+a=2a$
$-x+a-x-a=-2x$
$x-a+x+a=2x$





Gives 1 real solution

$-2x=10-x$
$x=10$ true all a

$2a=10-x$

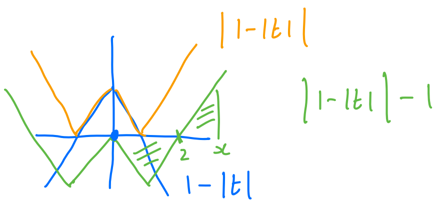
$-a \le x < a$
$10-a < 10-x \le 10+a$
$10-a < 2a \le 10+a$
$\frac{10}{3} < a \le 10$

$2x=10-x$
$x=\frac{10}{3}$

17. How many positive solutions for x are there for the equation:

$$\int_0^x \Big|1-|t|\Big|-1 \; dt = 0$$

A 0
B 1
C 2
D 4
E infinitely many



As x increases from 0 area is below x-axis

At some $x>2$, areas below + above are equal so integral will be zero

After this integral is positive for all x

18. How many values of k give 3 distinct real solutions to $|x-k| = x^2 - 4x + 4$ $= (x-2)^2$

A 0

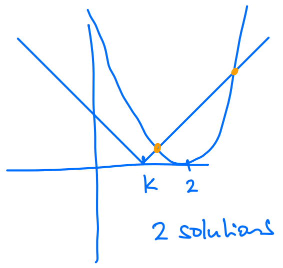
B 1

C 2

D 3

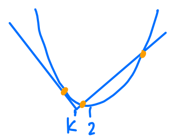
E 4

F infinitely many

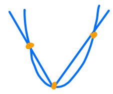


2 solutions

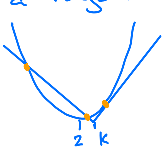
As k gets closer to 2, there comes a point when the line is a tangent to the curve which gives 3 solutions



At k = 2 3 solutions



Similarly when the line is a tangent on the other side of 2 3 solutions



19. Consider the inequality $m|x| + 2 < x^2 + 3x + 6$ where m is a real constant.

What is the complete set of values for m that make this inequality true for all real x?

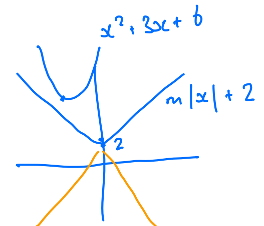
A $m < 1$

B $m < \dfrac{3}{2}$

C $m < 2$

D $-7 < m < 1$

E $-1 < m < 7$



$m|x|+2$ for $m<0$ always true

When line is a tangent

$2 - mx = x^2 + 3x + 6$

$x^2 + (m+3)x + 4 = 0$

$\Delta = 0$ $\quad (m+3)^2 - 16 = 0$

$m+3 = \pm 4$

$m = 1, -7$

$m < 1$

20. The equation $\big||x-1|-3\big| - 5 = k$ has exactly 3 solutions.

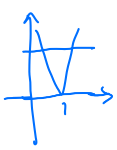
What is the value of k ?

A -2 B -1 C 1 D 3 E 5

$\big||x-1|-3\big| - 5 = k$ $\quad k+5 > 0$

$|x-1| - 3 = \pm(k+5)$ $\quad k > -8$

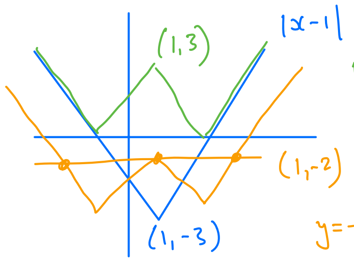
$|x-1| = k+8$, $-k-2$ $\quad k < -2$



For 3 solutions need one of these to be at the vertex

$k > -5$ so $k = -2$

or graphically (track vertex position)



y=-2 gives 3 solutions