

Mock TMUA Set B: Paper 1

20 questions

75 minutes

No calculator allowed

1. Find the value of $\int_1^2 6\sqrt{x} + \frac{16}{x^3} dx$

A $8\sqrt{2} - 10$

B $4\sqrt{8} - \frac{1}{4}$

C $8\sqrt{2} + 2$

D $4\sqrt{8} + \frac{1}{4}$

E $4\sqrt{2} + 6$

2. How many solutions does the following equation have in the range $0 \leq x \leq 2\pi$

$$\sin 2x + \sin^2 x = 1$$

A 2

B 3

C 4

D 6

E 8

F infinitely many

3. What is the shortest distance from the point A (5 , 0) to the curve with equation $y = x^2 + 1$

A $\sqrt{5}$

B $2\sqrt{2}$

C $2\sqrt{5}$

D 5

E $5\sqrt{2}$

4. Consider an arithmetic sequence with $(2n + 1)$ terms.

What is the ratio of the sum of odd terms to the sum of even terms?

- A $n^2 : n^2 - 1$
- B $n + 1 : n$
- C $n : n - 1$
- D $n + 1 : n - 1$
- E $n(n + 1) : 2n + 1$

5. Find the sum of the solutions of the equation $x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x$

- A - 6
- B - 2
- C 2
- D 6
- E 10

6. The function f is given by $f(x) = \left(\frac{1}{x} - \frac{2}{x^2}\right)^2$ $x \neq 0$
What is the value of $f''(1)$

- A - 6
- B - 2
- C 26
- D 38
- E 122

7. A sequence is defined by $u_n = \sum_{r=0}^{n-1} u_r$ and $u_0 = 1$

Evaluate $\sum_{r=0}^{\infty} \frac{1}{u_r}$

- A the sum does not converge
- B 1
- C 2
- D 3
- E $\frac{2}{3}$

8. Given that $3^a = 16$ and $2^b = 27$, find the value of ab

- A 3
- B $\frac{7}{2}$
- C 4
- D $\frac{9}{2}$
- E 12

9. In a set of k consecutive integers, the largest number is 23. What is the mean of the set?

- A $\frac{1}{2}(k + 45)$
- B $25 - 2k$
- C $\frac{23}{2}k$
- D $\frac{1}{2}(25 - k)$
- E $\frac{1}{2}(47 - k)$

10. Evaluate the following integral $\int_0^4 x|x-4| \, dx$

A $-\frac{32}{3}$

B $-\frac{16}{3}$

C $\frac{16}{3}$

D $\frac{32}{3}$

E 0

11. Which of the following is the largest?

A $2^{\frac{1}{2}}$

B $3^{\frac{1}{3}}$

C $4^{\frac{1}{4}}$

D $6^{\frac{1}{6}}$

E $12^{\frac{1}{12}}$

12. Two players take it in turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

A $\frac{3}{5}$

B $\frac{4}{7}$

C $\frac{5}{9}$

D $\frac{6}{11}$

E $\frac{7}{12}$

13. How many real roots does the equation $12x^5 - 45x^4 + 40x^3 - 10$ have?

- A 1 B 2 C 3 D 4 E 5

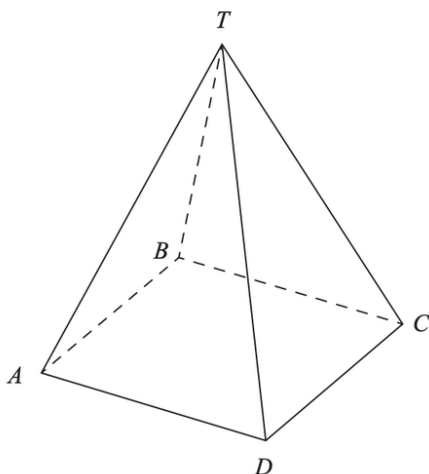
14. Find the coefficient of x^4y^2 in the expansion of $(1 + x^2 + y)^6$

- A 6
B 15
C 30
D 90
E 120

15. Find the minimum value of $f(x) = 25\sin^4x - 30\sin^2x + 11$

- A 1
B 2
C 5
D 8
E 11

16. A square based pyramid, with base ABCD, and vertex T has all edges of length 2m.
Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT.



- A $\sqrt{3} - 1$
 B 2
 C $\sqrt{2} + 1$
 D $\sqrt{4 + \sqrt{3}}$
 E $2\sqrt{2}$

17. Three geometric transformations are defined as follows:
 R is a reflection in the y -axis
 S is a stretch parallel to the x -axis, scale factor $1/2$
 T is a translation by 3 units in the negative x direction

These three transformations are applied to the graph of $y = \sqrt{x}$ resulting in the graph of $y = \sqrt{3 - 2x}$
 In which order were the transformations applied?

- A R then S then T
 B R then T then S
 C S then R then T
 D S then T then R
 E T then S then R

18. The curve C has equation $y = x^2 + bx + 3$ where $b \geq 0$
Find the value of b that minimises the distance between the origin and the stationary point of the curve C .
- A $b = 0$
B $b = 2$
C $b = \sqrt{6}$
D $b = \sqrt{10}$
E $b = 10$
19. How many solutions does the following equation have in the range $0 \leq x < 2\pi$
$$2 = \sin x + \sin^2 x + \sin^3 x + \sin^4 x + \dots$$
- A 0
B 1
C 2
D 3
E infinitely many
20. The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x .
Given that $\int_2^4 f(x) \, dx = A$ and $\int_4^6 f(x) \, dx = B$
evaluate $\int_{-2}^2 f(x+4) + 2 \, dx$ in terms of A and B
- A $A + B$
B $2(A + B)$
C $A + B + 2$
D $A + B + 8$
E $4(A + B) + 2$

Mock TMUA Set B: Paper 2

20 questions

75 minutes

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1. For how many values of the non-zero constant k , does the following equation have only one real solution

$$kx^2 - (k - 1)x + k = 0$$

- A no values of k
- B one value of k
- C two values of k
- D all values of k except $k = 1$
- E all values of k

2. How many solutions does the following equation have in the range $0 \leq x \leq \pi$

$$\sin 2x = \cos x$$

- A 1
- B 2
- C 3
- D 4
- E infinitely many

3. For which real numbers x does the following inequality hold

$$\frac{x}{x^2 + 1} \leq \frac{1}{2}$$

- A all real numbers x
- B only for real numbers $x \leq \frac{1}{2}$
- C only for real numbers $x \geq 1$
- D none of the above

4. Consider the following attempt at finding the number of solutions to the equation

$$6 \sin x = 7 \tan x \quad \text{for } 0 \leq x \leq 360.$$

I $6 \sin x = 7 \frac{\sin x}{\cos x}$

II $6 = \frac{7}{\cos x}$

III $\cos = \frac{6}{7}$

IV There are two real solutions to this equation.

Which statement describes this attempt?

- A It is completely correct
- B It is incorrect and the first mistake occurs on line I
- C It is incorrect and the first mistake occurs on line II
- D It is incorrect and the first mistake occurs on line III
- E It is incorrect and the first mistake occurs on line IV

5. The function f is defined for positive integers and satisfies

$$f(1) = 1 \text{ and } f(2n) = f(n) \text{ and } f(2n + 1) = f(n) + 1$$

What is the value of $f(9)$

- A 1
- B 2
- C 3
- D 4
- E 5

6. Consider the following two statements about the polynomial $p(x)$

P: $p(x)$ has at least one real root.

Q: $p(x)$ is a polynomial of order n , where n is an odd integer.

Which of the following is correct?

- A P is **necessary** and **sufficient** for Q
- B P is **not necessary** and **not sufficient** for Q
- C P is **necessary** but **not sufficient** for Q
- D P is **sufficient** but **not necessary** for Q

7. The real numbers a and b are such that exactly one of the following statements is true. Which is the true statement?

- A $a \geq 0$
- B $a < b$
- C $a^2 > b^2$
- D $|a| \leq |b|$

8. Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$.

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$

Which of the following is true?

- A The condition $p(a)p(b) < 0$ is **necessary** and **sufficient** for (*)
- B The condition $p(a)p(b) < 0$ is **not necessary** and **not sufficient** for (*)
- C The condition $p(a)p(b) < 0$ is **necessary** but **not sufficient** for (*)
- D The condition $p(a)p(b) < 0$ is **sufficient** but **not necessary** for (*)

9. A student wishes to evaluate the function $f(x) = \frac{\tan x}{x}$ where x is in radians, but only has a calculator that works in degrees.

What can the student type into their calculator in order to correctly evaluate $f(5)$

- A $\frac{1}{5} \times \tan(\pi \times 5 \div 180)$
- B $5 \times \tan(\pi \times 5 \div 180)$
- C $\tan(\pi \times 5 \div 180) \div (\pi \times 5 \div 180)$
- D $\tan(180 \times 5 \div \pi) \div (180 \times 5 \div \pi)$
- E $\frac{1}{5} \times \tan(180 \times 5 \div \pi)$
- F $5 \times \tan(180 \times 5 \div \pi)$

10. A sequence is such that $u_1 = 6$ and $u_2 = 3$ and $u_{n+1} = \frac{u_n}{u_{n-1}}$ for $n > 1$

What is the value of u_{2023} ?

- A $\frac{1}{2}$
- B $\frac{1}{6}$
- C 2
- D 3
- E 6

11. The fact that $5 \times 6 = 30$,
is a counter example to which of the following statements:

- A the product of any two odd integers is odd
- B if the product of two integers is a multiple of 4 then the integers are not consecutive
- C if the product of two integers is not a multiple of 4 then the integers are not consecutive
- D any even integer can be written as the product of two even integers.

12. What is the reflection of the point (3,4) in the line $3x + 4y = 50$

- A (9 , 12)
- B (12 , 9)
- C (6 , 8)
- D (8 , 6)
- E (12 , 16)
- F (16 , 12)

13. Consider the four options below about a particular statement:

- A The statement is true if $x^2 < 1$
- B The statement is true if and only if $x^2 < 1$
- C The statement is true if $x^2 < 2$
- D The statement is true if and only if $x^2 < 2$

Given that exactly one of these options is correct, which one is it?

14. Find the minimum value of $f(x) = 2x^3 - 9x^2 + 12x + 3$ for $0 \leq x \leq 2$

- A 1
- B 2
- C 3
- D 5
- E 7

15. Given that the numbers x and y satisfy $(x - 1)^2 + y^2 \leq 1$.

What is the largest value that $x + y$ can be?

- A 1
- B $\sqrt{2}$
- C 2
- D $1 + \sqrt{2}$
- E 4

16. Given that $x = -b$ is a root of the equation
 $f(x) = ax^3 + ax^2 + ax + b$ where a and b are constants.

Find the range of possible value of a .

- A There are no possible values of a
- B $a < 1$
- C $0 \leq a \leq \frac{4}{3}$
- D $a \geq 1$
- E a can be any real number

17. Let $a, b, c > 0$. The equations: $\log_b a = 2$ $\log_a(4c - 5) = 2$ $\log_b(c - 1) = 3$

- A specify a uniquely
- B are satisfied by two values of a
- C are satisfied by infinitely many solutions for a
- D are contradictory

18. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

A $\int_0^{8\pi} \sin^{64} x \, dx$

B $\int_0^{4\pi} 64(\cos^6 x - 1) \, dx$

C $\int_0^1 (x^2 - 2)\sin^6(\pi x) \, dx$

D $\int_0^{4\pi} (3 + \sin x)^6 \, dx$

19. Which of the following statements are true?

- I **There exists** a real y such that **for all** real x , $y > x$
- II **There exists** a real x such that **for all** real y , $x + y > xy$
- III **For all** real x , **there exists** real y such that $x - y = xy^2 + 1$

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F II and III only
- G I and III only
- H I, II and III

20. f is a function and a, b are real numbers.

Given that exactly one of the following statements is true, which one is it?

- A $f(a) \geq f(b)$ if and only if $a \geq b$
- B $f(a) \geq f(b)$ only if $a < b$
- C $f(a) < f(b)$ if $a \geq b$
- D $a \geq b$ if $f(a) \geq f(b)$
- E $a < b$ only if $f(a) \geq f(b)$
- F $a < b$ only if $f(a) < f(b)$