

Mock TMUA Set B: Paper 1

20 questions

75 minutes

No calculator allowed

1. Find the value of $\int_1^2 6\sqrt{x} + \frac{16}{x^3} dx$

A $8\sqrt{2} - 10$

B $4\sqrt{8} - \frac{1}{4}$

C $8\sqrt{2} + 2$

D $4\sqrt{8} + \frac{1}{4}$

E $4\sqrt{2} + 6$

$$\begin{aligned}
 &= \int_1^2 6x^{\frac{1}{2}} + 16x^{-3} dx \\
 &= \left[4x^{\frac{3}{2}} - 8x^{-2} \right]_1^2 \\
 &= (4\sqrt{8} - 2) - (4 - 8) \\
 &= 8\sqrt{2} + 2
 \end{aligned}$$

2. How many solutions does the following equation have in the range $0 \leq x \leq 2\pi$

$\sin 2x + \sin^2 x = 1$

$\sin 2x = 1 - \sin^2 x$

A 2

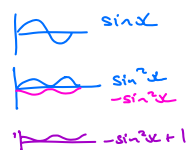
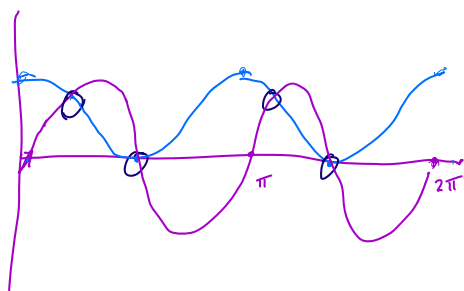
B 3

C 4

D 6

E 8

F infinitely many



3. What is the shortest distance from the point A (5, 0) to the curve with equation $y = x^2 + 1$

A $\sqrt{5}$

B $2\sqrt{2}$

C $2\sqrt{5}$

D 5

E $5\sqrt{2}$

$$\begin{aligned}
 \frac{dy}{dx} &= 2x \\
 \text{At B } \frac{dy}{dx} &= 2p
 \end{aligned}$$

$$\begin{aligned}
 y - 0 &= \frac{-1}{2p} (x - 5) \\
 y &= \frac{1}{2p} (5 - x)
 \end{aligned}$$

At B $p^2 + 1 = \frac{1}{2p} (5 - p)$

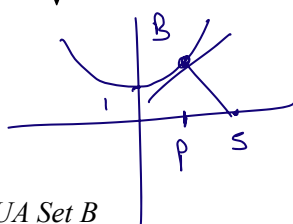
$2p^3 + 2p + p - 5 = 0$

$2p^3 + 3p - 5 = 0$

$(p-1)(2p^2 + 2p + 5)$

$p = 1$ no soln

B (1, 2) (5, 0)



$$\begin{aligned}
 &\sqrt{4^2 + 2^2} \\
 &= \sqrt{20} = 2\sqrt{5}
 \end{aligned}$$

4. Consider an arithmetic sequence with $(2n + 1)$ terms.

What is the ratio of the sum of odd terms to the sum of even terms?

A $n^2 : n^2 - 1$

☒ B $n + 1 : n$

C $n : n - 1$

D $n + 1 : n - 1$

E $n(n + 1) : 2n + 1$

$$a \quad a+d \quad a+2d \quad \dots \quad a+2nd$$

$$a+d \quad a+3d \quad \dots \quad a+(2n-1)d$$

$$a \quad a+2d \quad a+4d \quad \dots \quad a+2nd$$

$$S_n = \frac{1}{2} (a+d + a+(2n-1)d)$$

$$S_{n+1} = \frac{n+1}{2} (a+a+2nd)$$

$$= n(a+nd)$$

$$= (n+1)(a+nd)$$

5. Find the sum of the solutions of the equation

$$x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x$$

☒ A -6

B -2

C 2

D 6

E 10

$$x^2 + 6x + 2\sqrt{x^2 + 6x} - 24 = 0$$

$$\text{Let } y = \sqrt{x^2 + 6x}$$

$$\sqrt{x^2 + 6x} = 4$$

$$y^2 + 2y - 24 = 0$$

$$x^2 + 6x - 16 = 0$$

$$(y-4)(y+6) = 0$$

$$(x+8)(x-2) = 0$$

$$x = -8, 2$$

6. The function f is given by

$$f(x) = \left(\frac{1}{x} - \frac{2}{x^2}\right)^2 \quad x \neq 0$$

What is the value of $f''(1)$

A -6

B -2

C 26

☒ D 38

E 122

$$= \frac{1}{x^2} - \frac{4}{x^3} + \frac{4}{x^4}$$

$$= x^{-2} - 4x^{-3} + 4x^{-4}$$

$$f'(x) = -2x^{-3} + 12x^{-4} - 16x^{-5}$$

$$f''(x) = 6x^{-4} - 48x^{-5} + 80x^{-6}$$

$$f''(1) = 6 - 48 + 80$$

$$= 38$$

7. A sequence is defined by $u_n = \sum_{r=0}^{n-1} u_r$ and $u_0 = 1$

Evaluate $\sum_{r=0}^{\infty} \frac{1}{u_r}$

$$\begin{aligned} u_0 &= 1 \\ u_1 &= u_0 = 1 \\ u_2 &= u_0 + u_1 = 2 \\ u_3 &= u_0 + u_1 + u_2 = 4 \end{aligned}$$

A the sum does not converge

B 1

C 2

D 3

E $\frac{2}{3}$

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{1}{u_k} &= \frac{1}{1} + \frac{1}{1} + \underbrace{\frac{1}{2} + \frac{1}{4} + \dots}_{=1} \\ &= 3 \end{aligned}$$

8. Given that $3^a = 16$ and $2^b = 27$, find the value of ab

A 3

B $\frac{7}{2}$

C 4

D $\frac{9}{2}$

E 12

$$\begin{aligned} 3^a &= 2^4 & 2^b &= 3^3 \\ a \log 3 &= 4 \log 2 & b \log 2 &= 3 \log 3 \\ ab &= \frac{4 \log 2}{\log 3} \times \frac{3 \log 3}{\log 2} = 12 \end{aligned}$$

9. In a set of k consecutive integers, the largest number is 23. What is the mean of the set?

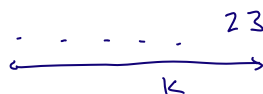
A $\frac{1}{2}(k + 45)$

B $25 - 2k$

C $\frac{23}{2}k$

D $\frac{1}{2}(25 - k)$

E $\frac{1}{2}(47 - k)$



$$\begin{aligned} \text{1st term is } 23 - k + 1 &= 24 - k \\ \text{Mean} &= \frac{1}{2}(24 - k + 23) = \frac{1}{2}(47 - k) \end{aligned}$$

10. Evaluate the following integral $\int_0^4 x|x-4| dx$

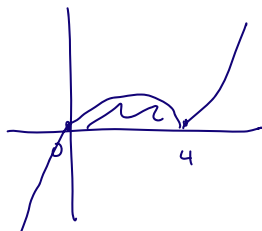
A $-\frac{32}{3}$

B $-\frac{16}{3}$

C $\frac{16}{3}$

D $\frac{32}{3}$

E 0



$$\int_0^4 4x - x^2 dx = \left[2x^2 - \frac{1}{3}x^3 \right]_0^4 = \left(32 - \frac{64}{3} \right) - (0) = \frac{32}{3}$$

11. Which of the following is the largest?

A $2^{\frac{1}{2}}$

B $3^{\frac{1}{3}}$

C $4^{\frac{1}{4}}$

D $6^{\frac{1}{6}}$

E $12^{\frac{1}{12}}$

$$\begin{array}{ccccc} 2^{6/12} & 3^{4/12} & 4^{3/12} & 6^{2/12} & 12^{1/12} \\ 64^{1/12} & 81^{1/12} & 64^{1/12} & 36^{1/12} & \end{array}$$

12. Two players take it in turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

A $\frac{3}{5}$

B $\frac{4}{7}$

C $\frac{5}{9}$

D $\frac{6}{11}$

E $\frac{7}{12}$

$$\frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \dots$$

$$a = \frac{1}{6} \quad r = \frac{25}{36} \quad S_{\infty} = \frac{1/6}{1/36} = \frac{6}{11}$$

13. How many real roots does the equation $12x^5 - 45x^4 + 40x^3 - 10$ have?

- A 1 B 2 C 3 D 4 E 5

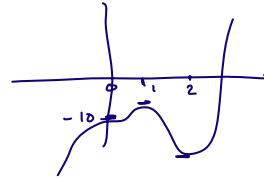
$$\frac{d}{dx} = 60x^4 - 180x^3 + 120x^2 = 0$$

$$x^4 - 3x^3 + 2x^2 = 0$$

$$(x-2)(x-1)x^2 = 0$$

$$x = 0, 1, 2$$

$$(0, 10) \text{ repeated } (1, -3) \quad (2, -26)$$



14. Find the coefficient of x^4y^2 in the expansion of $(1 + x^2 + y)^6$

- A 6
B 15
C 30
D 90
E 120

$$\binom{6}{r} (1+x^2)^r y^{6-r} \quad r=4$$

$$15 (1+x^2)^4 y^2$$

$$(1+x^2)^4$$

$$\binom{4}{2} (x^2)^2$$

$$6x^4$$

$$15 \times 6 = 90$$

$$\frac{6!}{4!2!} \binom{6}{4} (1+y)^4 (x^2)^2$$

$$(1+y)^4 = 1 + 4y + 6y^2 + \dots$$

$$15 \times 6 = 90$$

$$\frac{6!}{2!2!2!} \binom{6}{2} (x^2)^2 (y)^2 = \frac{6 \times 5 \times 4 \times 3}{2 \times 2} = 90$$

(Trinomial)

15. Find the minimum value of $f(x) = 25\sin^4x - 30\sin^2x + 11$

- A 1
B 2
C 5
D 8
E 11

$$\text{Let } y = \sin^2x$$

$$25y^2 - 30y + 11$$

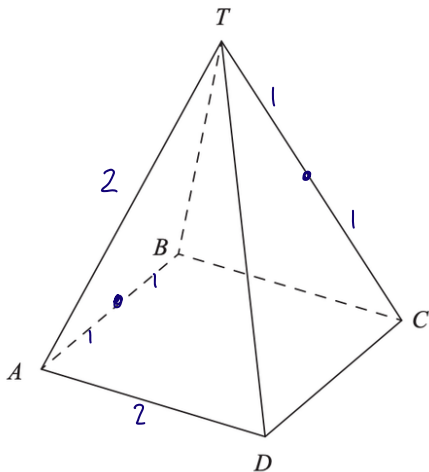
$$25\left(y^2 - \frac{6}{5}y\right) + 11$$

$$25\left(y - \frac{3}{5}\right)^2 - 25\left(\frac{9}{25}\right) + 11$$

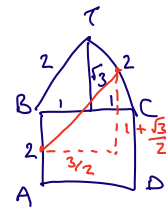
$$25\left(y - \frac{3}{5}\right)^2 + \underline{\underline{2}}$$

16. A square based pyramid, with base ABCD, and vertex T has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT.

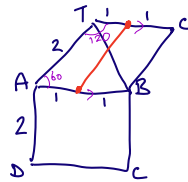


- A $\sqrt{3} - 1$
 B 2
 C $\sqrt{2} + 1$
 D $\sqrt{4 + \sqrt{3}}$
 E $2\sqrt{2}$



$$\left(\frac{3}{2}\right)^2 + \left(1 + \frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4} + 1 + \sqrt{3} + \frac{3}{4} = 4 + \sqrt{3}$$

$$CT = \sqrt{4 + \sqrt{3}}$$



CT = 2 shortest distance

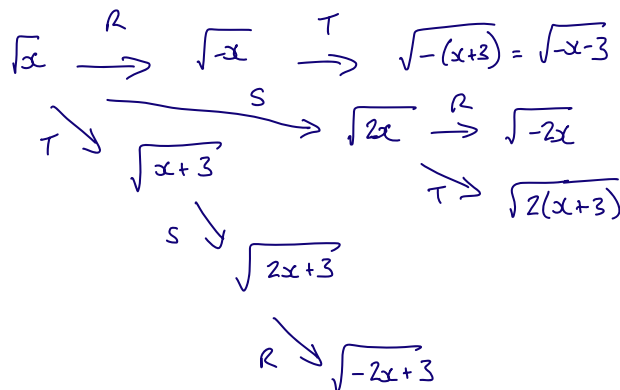
17. Three geometric transformations are defined as follows:

- R is a reflection in the y -axis $f(-x)$
 S is a stretch parallel to the x -axis, scale factor $1/2$ $f(2x)$
 T is a translation by 3 units in the negative x direction $f(x+3)$

These three transformations are applied to the graph of $y = \sqrt{x}$ resulting in the graph of $y = \sqrt{3 - 2x}$

In which order were the transformations applied?

- A R then S then T
 B R then T then S
 C S then R then T
 D S then T then R
 E T then S then R



18. The curve C has equation $y = x^2 + bx + 3$ where $b \geq 0$

Find the value of b that minimises the distance between the origin and the stationary point of the curve C .

A $b = 0$

B $b = 2$

C $b = \sqrt{6}$

D $b = \sqrt{10}$

E $b = 10$

$$y = x^2 + bx + 3$$

$$= \left(x + \frac{1}{2}b\right)^2 + 3 - \frac{1}{4}b^2$$

$$\text{T.P. } \left(-\frac{1}{2}b, 3 - \frac{1}{4}b^2\right)$$

$$\text{Dist}^2 = \frac{b^2}{4} + 9 - \frac{3}{2}b^2 + \frac{1}{16}b^4$$

$$\frac{1}{4} - \frac{3b}{4} = -\frac{5}{4}$$

$$= \frac{1}{16}(b^4 - 20b^2 + 144)$$

$$= \frac{1}{16}[(b^2 - 10)^2 + 44]$$

$$\text{Min when } b^2 = 10 \quad b = \sqrt{10}$$

19. How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$2 = \sin x + \sin^2 x + \sin^3 x + \sin^4 x + \dots$$

A 0

B 1

C 2

D 3

E infinitely many

$$\text{G.P. } a = \sin x$$

$$r = \sin x$$

$$S_{\infty} = \frac{\sin x}{1 - \sin x} = 2$$

$$\sin x = 2 - 2\sin x$$

$$3\sin x = 2$$

$$\sin x = \frac{2}{3} \quad 2 \text{ solutions}$$

20. The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x .

Given that $\int_2^4 f(x) dx = A$ and $\int_4^6 f(x) dx = B$

evaluate $\int_{-2}^2 f(x+4) + 2 dx$ in terms of A and B

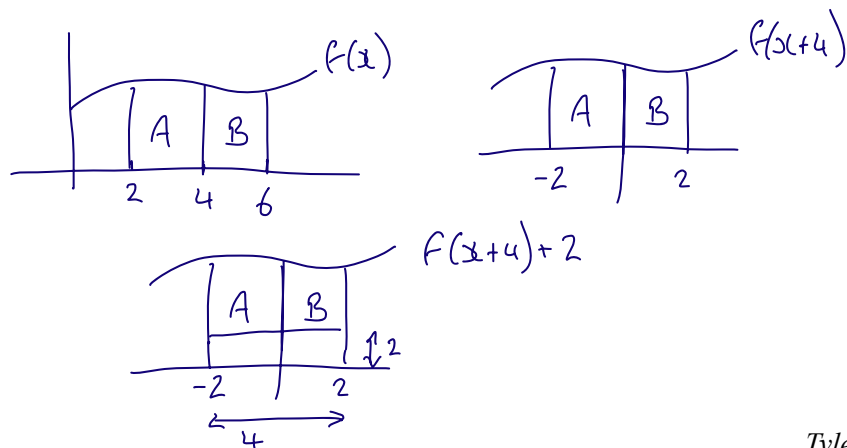
A $A + B$

B $2(A + B)$

C $A + B + 2$

D $A + B + 8$

E $4(A + B) + 2$



Mock TMUA Set B: Paper 2

20 questions

75 minutes

No calculator allowed

1. For how many values of the non-zero constant k , does the following equation have only one real solution

$$kx^2 - (k-1)x + k = 0$$

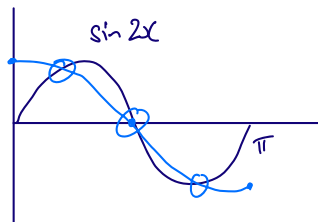
- A no values of k
B one value of k
☒ C two values of k
D all values of k except $k = 1$
E all values of k

$$\Delta = (k-1)^2 - 4k^2 = 0$$
$$k-1 = 2k \quad k-1 = -2k$$
$$k = -1 \quad k = \frac{1}{3}$$

2. How many solutions does the following equation have in the range $0 \leq x \leq \pi$

$$\sin 2x = \cos x$$

- A 1
B 2
☒ C 3
D 4
E infinitely many



3. For which real numbers x does the following inequality hold

$$\frac{x}{x^2 + 1} \leq \frac{1}{2}$$

- ☒ A all real numbers x
B only for real numbers $x \leq \frac{1}{2}$
C only for real numbers $x \geq 1$
D none of the above

$$2x \leq x^2 + 1$$
$$x^2 - 2x + 1 \geq 0$$
$$(x-1)^2 \geq 0$$

4. Consider the following attempt at finding the number of solutions to the equation

$$6 \sin x = 7 \tan x \quad \text{for } 0 \leq x \leq 360.$$

I $6 \sin x = 7 \frac{\sin x}{\cos x}$

II $6 = \frac{7}{\cos x}$ or $\sin x = 0$

III $\cos = \frac{6}{7}$ 2nd mistake ($\cos x = \frac{7}{6}$)

IV There are two real solutions to this equation.

Which statement describes this attempt?

A It is completely correct

B It is incorrect and the first mistake occurs on line I

☒ C It is incorrect and the first mistake occurs on line II

D It is incorrect and the first mistake occurs on line III

E It is incorrect and the first mistake occurs on line IV

5. The function f is defined for positive integers and satisfies

$$f(1) = 1 \text{ and } f(2n) = f(n) \text{ and } f(2n + 1) = f(n) + 1$$

What is the value of $f(9)$

A 1

☒ B 2 $f(1) = 1$ $f(2) = 1$ $f(3) = 2$ $f(4) = 1$

C 3 $f(5) = 2$ $f(6) = 2$ $f(7) = 3$ $f(8) = 1$

D 4 $f(9) = 2$

E 5

6. Consider the following two statements about the polynomial $p(x)$

P: $p(x)$ has at least one real root.

Q: $p(x)$ is a polynomial of order n , where n is an odd integer.

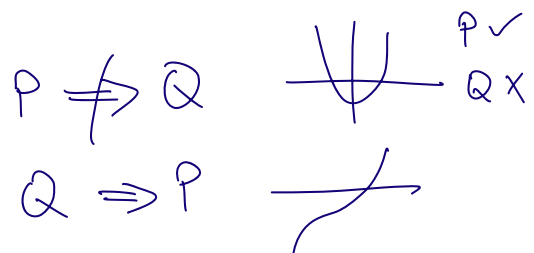
Which of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

☒ C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q



7. The real numbers a and b are such that exactly one of the following statements is true. Which is the true statement?

eg $a = -\frac{1}{2}, b = -1$

A	$a \geq 0$	if A is only true statement ✓	X
B	$a < b$	$b \leq a$	X
C	$a^2 > b^2$	$b^2 \geq a^2$ $(b-a)(b+a) \geq 0$ $b \leq -a$	X
D	$ a \leq b $	$ a > b $ contradiction so not A X $\Rightarrow a < 0$	✓

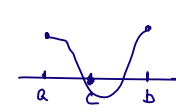
if B true $a < b$
 C false $b \geq -a$ so $b > 0$
 $|b| \geq |a|$ D also true X

$\therefore a < 0, a \geq b \Rightarrow b < 0$
 $\Rightarrow b^2 \geq a^2$ so C false
 $\therefore D$ is true

8. Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$.

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$

Which of the following is true?

- A The condition $p(a)p(b) < 0$ is **necessary** and **sufficient** for (*)
- B The condition $p(a)p(b) < 0$ is **not necessary** and **not sufficient** for (*)
- C The condition $p(a)p(b) < 0$ is **necessary** but **not sufficient** for (*)
- D The condition $p(a)p(b) < 0$ is **sufficient** but **not necessary** for (*)
- $p(a)p(b) < 0 \Rightarrow$ function crosses x -axis (sufficient)
 $\Rightarrow$ * true
- * true $\nRightarrow p(a)p(b) < 0$ eg  (not necessary)

9. A student wishes to evaluate the function $f(x) = \frac{\tan x}{x}$ where x is in radians, but only has a calculator that works in degrees.

What can the student type into their calculator in order to correctly evaluate $f(5)$

- A $\frac{1}{5} \times \tan(\pi \times 5 \div 180)$
- B $5 \times \tan(\pi \times 5 \div 180)$
- C $\tan(\pi \times 5 \div 180) \div (\pi \times 5 \div 180)$
- D $\tan(180 \times 5 \div \pi) \div (180 \times 5 \div \pi)$
- E $\frac{1}{5} \times \tan(180 \times 5 \div \pi)$
- F $5 \times \tan(180 \times 5 \div \pi)$

$$f(5) = \frac{\tan\left(\frac{180 \times 5}{\pi}\right)}{5}$$

10. A sequence is such that $u_1 = 6$ and $u_2 = 3$ and $u_{n+1} = \frac{u_n}{u_{n-1}}$ for $n > 1$

What is the value of u_{2023} ?

- A $\frac{1}{2}$
 B $\frac{1}{6}$
 C 2
 D 3
 E 6

$$u_3 = \frac{3}{6} = \frac{1}{2}$$

$$u_4 = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

$$u_5 = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$u_6 = \frac{\frac{1}{3}}{\frac{1}{6}} = 2$$

$$u_7 = \frac{2}{\frac{1}{3}} = 6 = u_1 = u_{2023}$$

sequence repeats with period 6

$$\begin{array}{r} 337 \div 6 \\ 6 \overline{) 2022} \\ \underline{12} \\ 80 \\ \underline{48} \\ 320 \\ \underline{300} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

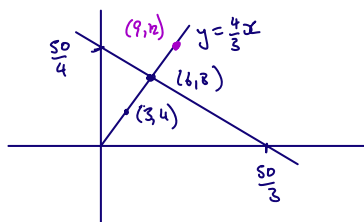
11. The fact that $5 \times 6 = 30$,
 is a counter example to which of the following statements:

- A the product of any two odd integers is odd
 B if the product of two integers is a multiple of 4 then the integers are not consecutive
 C if the product of two integers is not a multiple of 4 then the integers are not consecutive
 D any even integer can be written as the product of two even integers.

12. What is the reflection of the point (3,4) in the line $3x + 4y = 50$

- A (9, 12)
 B (12, 9)
 C (6, 8)
 D (8, 6)
 E (12, 16)
 F (16, 12)

$$y = -\frac{3}{4}x + \frac{25}{2}$$



need normal passing through (3,4)

$$y = \frac{4}{3}x$$

$$\frac{4}{3}x + \frac{3}{4}x = \frac{25}{2}$$

$$\frac{25x}{12} = \frac{25}{2}$$

$$x = 6 \quad y = 8$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} 6 \\ 8 \end{pmatrix} \rightarrow \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

13. Consider the four options below about a particular statement:

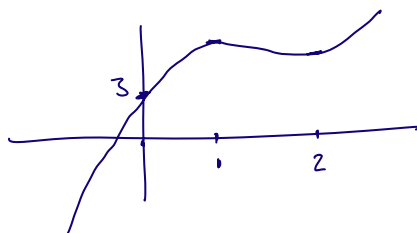
- ☒ A The statement is true if $x^2 < 1$
☐ B The statement is true if and only if $x^2 < 1$ $B \Rightarrow A$
☐ C The statement is true if $x^2 < 2$ $C \Rightarrow A$
☐ D The statement is true if and only if $x^2 < 2$ $D \Rightarrow C$

Given that exactly one of these options is correct, which one is it?

14. Find the minimum value of $f(x) = 2x^3 - 9x^2 + 12x + 3$ for $0 \leq x \leq 2$

- ☐ A 1
☐ B 2
☒ C 3
☐ D 5
☐ E 7

$$\begin{aligned}
 f(0) &= 3 & f'(x) &= 6x^2 - 18x + 12 = 0 \\
 f(1) &= 8 & x^2 - 3x + 2 &= 0 \\
 f(2) &= 16 - 36 + 27 = 7 & (x-1)(x-2) &= 0
 \end{aligned}$$

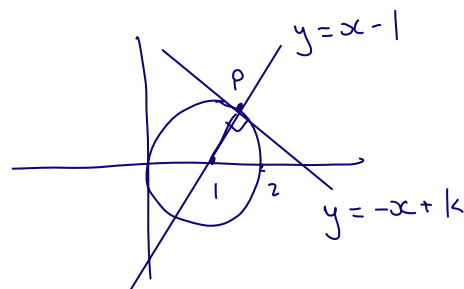


15. Given that the numbers x and y satisfy $(x-1)^2 + y^2 \leq 1$.

What is the largest value that $x + y$ can be?

- ☐ A 1
☐ B $\sqrt{2}$
☐ C 2
☒ D $1 + \sqrt{2}$
☐ E 4

$$\begin{aligned}
 \text{Max } x+y &= k \\
 y &= -x+k
 \end{aligned}$$



$$\begin{aligned}
 y &= x-1 & \frac{1}{\sqrt{2}} &= -1 - \frac{1}{\sqrt{2}} + k \\
 \text{At P: } 2(x-1)^2 &= 1 & k &= 1 + \sqrt{2} \\
 (x-1) &= \pm \frac{1}{\sqrt{2}} \\
 x &= 1 \pm \frac{1}{\sqrt{2}} \\
 y &= \pm \frac{1}{\sqrt{2}}
 \end{aligned}$$

16. Given that $x = -b$ is a root of the equation
 $f(x) = ax^3 + ax^2 + ax + b$ where a and b are constants.

Find the range of possible value of a .

A There are no possible values of a

B $a < 1$

C $0 \leq a \leq \frac{4}{3}$

D $a \geq 1$

E a can be any real number

$$ax^3 + ax^2 + ax + b = 0 \quad x = -b \text{ is root}$$

$$-ab^3 + ab^2 - ab + b = 0$$

$$\text{ii) } b \neq 0 \quad -ab^2 + ab + (1-a) = 0$$

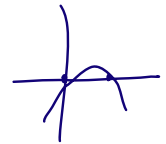
$$ab^2 - ab + (a-1) = 0 \quad \text{need a solution}$$

$$\Delta \geq 0 \quad a^2 - 4a(a-1) \geq 0$$

$$4a - 3a^2 \geq 0$$

$$a(4-3a) \geq 0$$

$$0 \leq a \leq \frac{4}{3}$$



17. Let $a, b, c > 0$. The equations: $\log_b a = 2$ $\log_a(4c - 5) = 2$ $\log_b(c - 1) = 3$

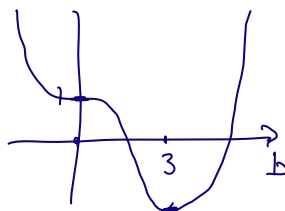
A specify a uniquely

B are satisfied by two values of a

C are satisfied by infinitely many solutions for a

D are contradictory

$$\begin{aligned} \text{1. P. } 4b^3 - 12b^2 &= 0 \\ 4b^2(b-3) &= 0 \\ b &= 0, 3 \\ 3^4 - 4(3^3) + 1 &< 0 \end{aligned}$$



$$\begin{aligned} b^2 &= a & a^2 &= 4c - 5 & b^3 &= c - 1 \\ b^4 &= a^2 & 4c &= a^2 + 5 & 4c &= 4b^3 + 4 \end{aligned}$$

$$b^4 + 5 = 4b^3 + 4$$

$$b^4 - 4b^3 + 1 = 0$$

2 real solutions for b
 $\Rightarrow$ 2 solutions for a

18. Which of the following integrals has the largest value?
 You are not expected to calculate the exact values of any of these.

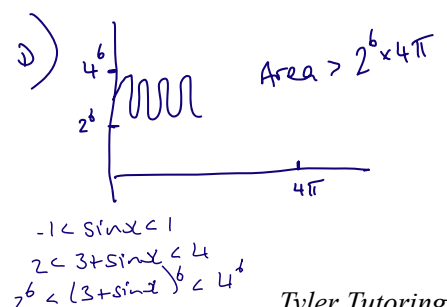
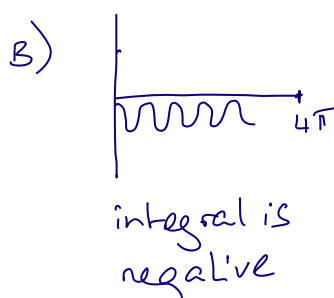
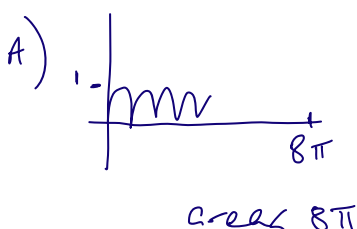
A $\int_0^{8\pi} \sin^{64} x \, dx$

B $\int_0^{4\pi} 64(\cos^6 x - 1) \, dx$

C $\int_0^1 (x^2 - 2)\sin^6(\pi x) \, dx$

D $\int_0^{4\pi} (3 + \sin x)^6 \, dx$

c) $x^2 - 2$
 negative from 0-1
 $\sin^6(\pi x) > 0$
 so integral is negative



19. Which of the following statements are true?

- I **There exists** a real y such that **for all** real x , $y > x$
- II **There exists** a real x such that **for all** real y , $x + y > xy$
- III **For all** real x , **there exists** real y such that $x - y = xy^2 + 1$
- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F** II and III only
- G I and III only
- H I, II and III

False - There is no largest number

True $x = 1$

True if $\Delta \geq 0$

$$\begin{aligned} \text{III} \quad x - y &= xy^2 + 1 \\ xy^2 + y + 1 - x &= 0 \\ \Delta &= 1 - 4x(1-x) = 1 - 4x + 4x^2 \\ &= (2x-1)^2 \geq 0 \\ \Rightarrow &\text{always a solution for } y \end{aligned}$$

20. f is a function and a, b are real numbers.

Given that exactly one of the following statements is true, which one is it?

- A $f(a) \geq f(b)$ if and only if $a \geq b$ if $f(a) \geq f(b)$ then $a \geq b$ if $a \geq b$ then $f(a) \geq f(b)$ * same
- B $f(a) \geq f(b)$ only if $a < b$ if $f(a) \geq f(b)$ then $a < b$] contrapositive
- C $f(a) < f(b)$ if $a \geq b$ if $a \geq b$ then $f(a) < f(b)$
- D $a \geq b$ if $f(a) \geq f(b)$ if $f(a) \geq f(b)$ then $a \geq b$] *
- E** $a < b$ only if $f(a) \geq f(b)$ if $a < b$ then $f(a) \geq f(b)$] contrapositive
- F $a < b$ only if $f(a) < f(b)$ if $a < b$ then $f(a) < f(b)$]

$A \Rightarrow B$
not $B \Rightarrow$ not A

A, D equivalent

B, C D, F contrapositive