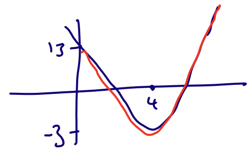
1. Sketch each of the following functions and find the range for the given domains:

a) $f(x) = x^2 - 8x + 13 \qquad x \in \mathbb{R} \quad x > 0$

$= (x-4)^2 - 3$

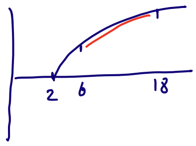


$f(x) \geq -3$

b) $f(x) = \sqrt{x-2} \qquad x \in \mathbb{R} \quad 6 < x < 18$

$x = 6 \quad f(x) = 2$

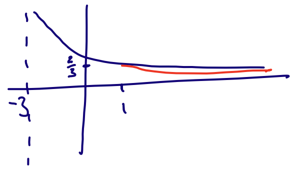
$x = 18 \quad f(x) = 4$



$2 < f(x) < 4$

c) $f(x) = \dfrac{2}{x+3} \qquad x \in \mathbb{R} \quad x \geq 1$

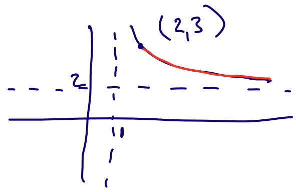
$x = 1 \quad f(x) = \frac{1}{2}$



$0 < f(x) \leq \frac{1}{2}$

d) $f(x) = \dfrac{1}{x-1} + 2 \qquad x \in \mathbb{R} \quad x > 2$

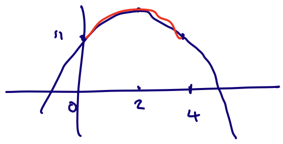
$x = 2 \quad f(x) = 3$



$2 < f(x) < 3$

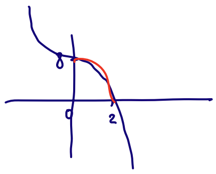
e) $f(x) = 15 - (x-2)^2 \qquad x \in \mathbb{R} \quad 0 \leq x \leq 4$

$f(0) = 11 \quad f(4) = 11$



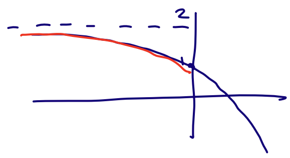
$11 \leq f(x) \leq 15$

f) $f(x) = 8 - x^3 \qquad x \in \mathbb{R} \quad 0 \leq x \leq 2$



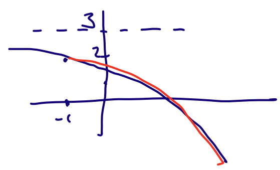
$0 \leq f(x) \leq 8$

g) $f(x) = 2 - e^x \qquad x \in \mathbb{R} \quad x \leq 0$



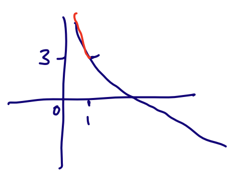
$1 \leq f(x) < 2$

h) $f(x) = 3 - e^{x+1} \qquad x \in \mathbb{R} \quad x \geq -1$



$f(x) \leq 2$

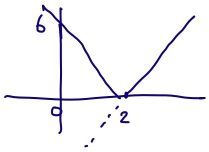
i) $f(x) = 3 - \ln x \qquad x \in \mathbb{R} \quad 0 < x < 1$



$f(x) > 3$

2. Sketch each of the following graphs, stating any values of x for which the function is not defined

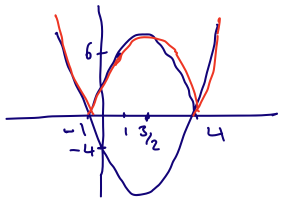
a) $y = |3x - 6|$



b) $y = |x^2 - 3x - 4|$

$(x - \frac{3}{2})^2 - \frac{9}{4} - 4$

$f(-1) = 0$ $f(4) = 0$



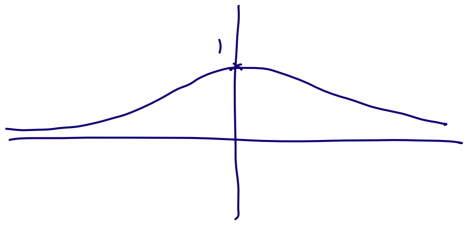
c) $y = \dfrac{1}{1 + x^2}$

Max when $(1+x^2)$ is min $(=1)$

As $x \to \pm\infty$ $y \to 0$

Even function

$(0, 1)$



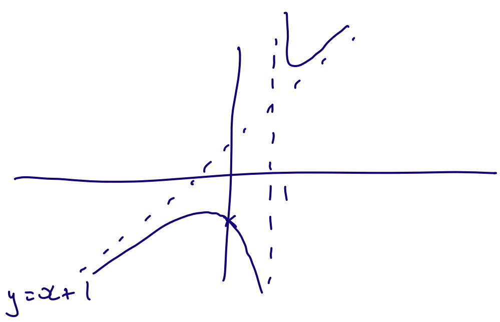
d) $y = \dfrac{x^2 + 1}{x - 1}$ $= x + 1 + \dfrac{2}{x-1}$

Asymptote $x = 1$

$(0, -1)$

Asymptote at $y = x + 1$

not defined for $x = 1$



e) $y^2 = x^3$

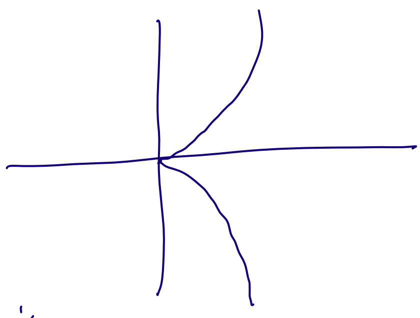
$x \geq 0$

symmetry in x-axis

$(0, 0)$

Stat pt at $x = 0$

not defined for $x < 0$

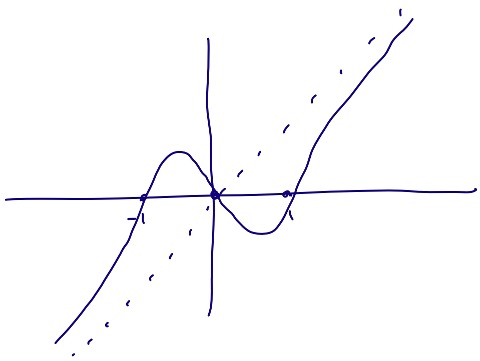


f) $y = \sqrt[3]{x^3 - x}$

odd function

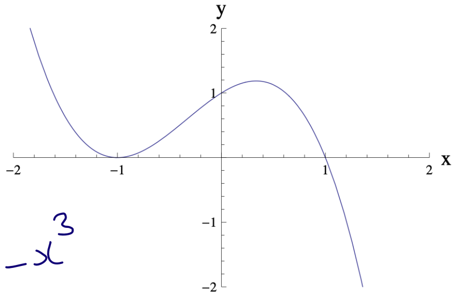
$y = 0$ $x = 0$ $x = \pm 1$

$x \to \infty$ $y \to x$



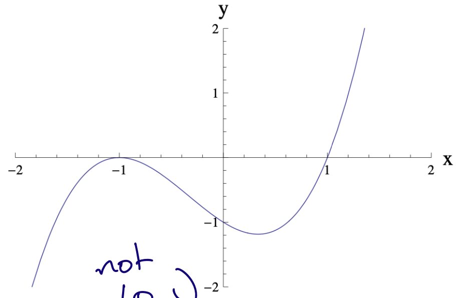
+ve cubic, $(0,1)$, $(1,0)$

3a) A sketch of the graph $y = x^3 - x^2 - x + 1$ appears on which of the following axes?



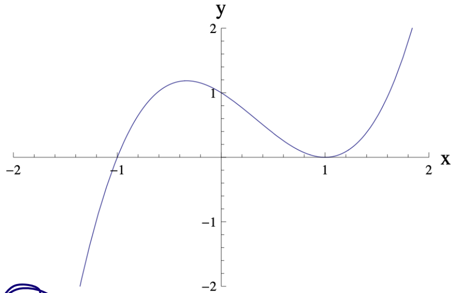
$-x^3$

(a)

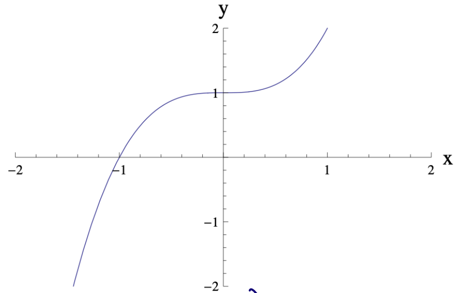


not $(0,1)$

(b)



(c)



(d) not $(1,0)$

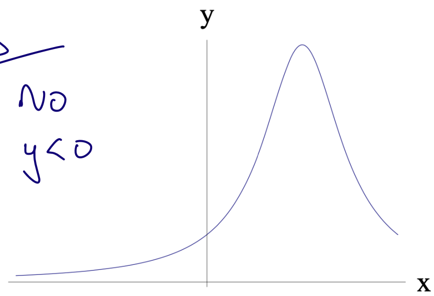
b) Which of the following graphs is a sketch of $y = \dfrac{1}{6x - x^2 - 10}$ $= -\dfrac{1}{x^2 - 6x + 10}$

$= -\dfrac{1}{(x-3)^2+1} < 0$

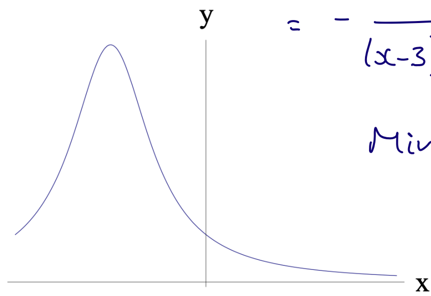
Min at $x = 3$

a, b

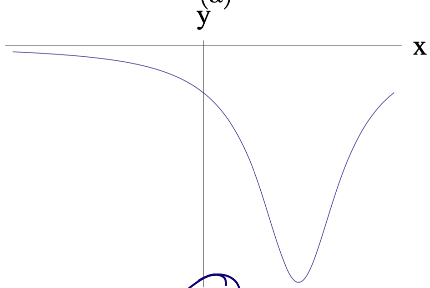
No $y<0$



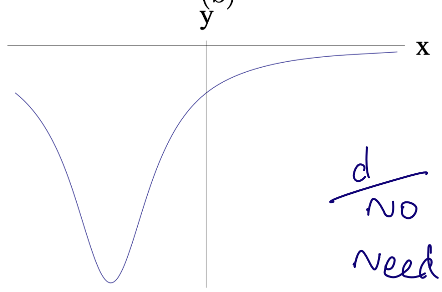
(a)



(b)



(c)



(d)

d

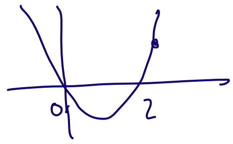
No

Need min at $x = 3$

4. Find the composite function $fg(x)$ and sketch this function.
State any values of x for which the function $fg(x)$ is not valid.

a) $f(x) = x^2 - 4$ $\qquad$ $g(x) = 2x - 2$

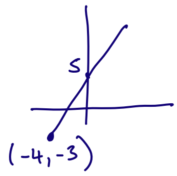
$$fg(x) = (2x-2)^2 - 4$$
$$= 4x^2 - 8x$$
$$= 4x(x-2)$$



b) $f(x) = 2x^2 - 3$ $\qquad$ $g(x) = \sqrt{x+4}$

$x \geqslant -4$

$$fg(x) = 2(x+4) - 3$$
$$= 2x + 5$$

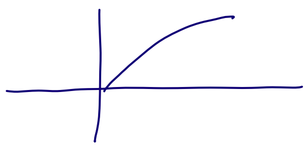


Not valid $x < -4$

c) $f(x) = 2e^{\frac{1}{2}x}$ $\qquad$ $g(x) = ln(4x)$

$x > 0$

$$fg(x) = 2e^{\frac{1}{2}\ln(4x)}$$
$$= 2(4x)^{1/2} = 4\sqrt{x}$$



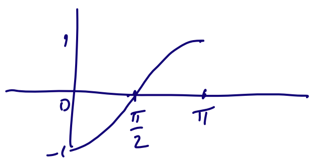
Not valid for $x \leq 0$

d) $f(x) = sin\,x$ $\qquad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ $\qquad$ $g(x) = x - \frac{\pi}{2} \quad x \geq 0$

Domain of $g(x)$

$x \geq 0 \Rightarrow g(x) \geq -\frac{\pi}{2}$

$$fg(x) = \sin\left(x - \frac{\pi}{2}\right)$$



Domain of $f(x)$

$g(x) \leq \frac{\pi}{2} \Rightarrow x \leq \pi$

Combined domain $0 \leq x \leq \pi$

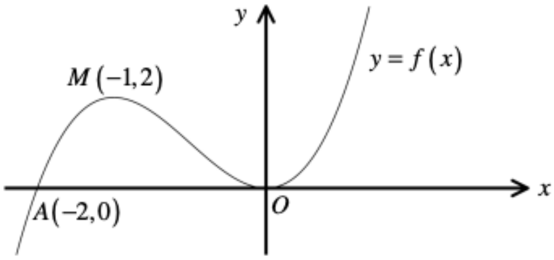
Defined for

$0 \leq x \leq \pi$

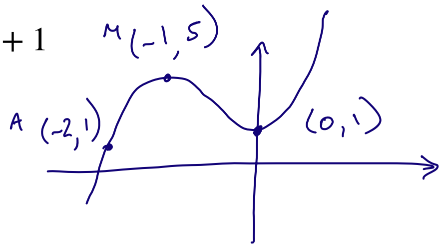
So range is $-1 \leq fg(x) \leq 1$

5. The figure shows the graph of the curve with equation $y = f(x)$

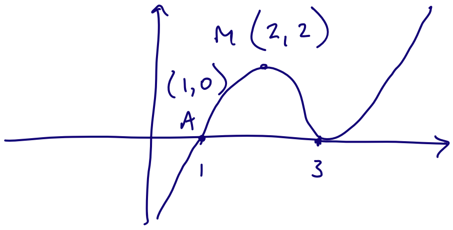
Sketch the graphs of the following functions and include the new coordinates of points A and M.



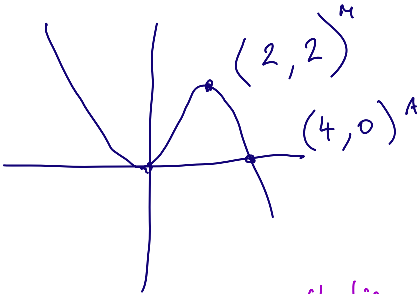
a) $y = 2f(x) + 1$



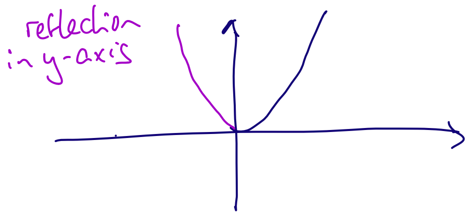
b) $y = f(x - 3)$



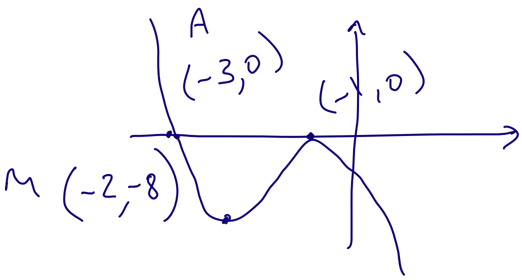
c) $y = f(-\frac{1}{2}x)$



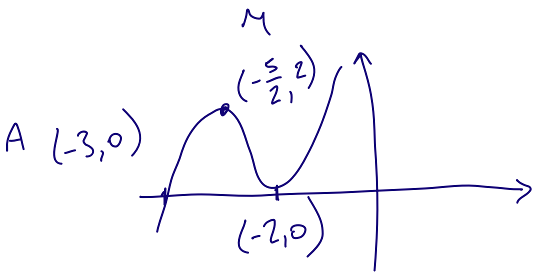
d) $y = f(|x|)$



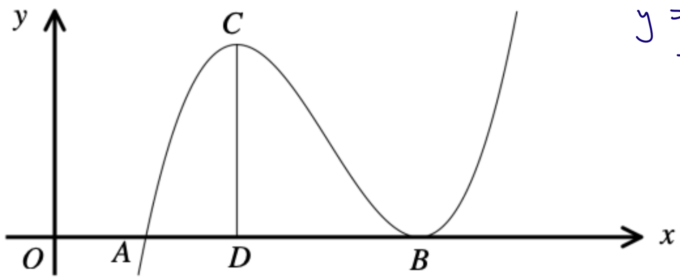
e) $y = -4f(x + 1)$



f) $y = f(2x + 4)$



6. The figure shows a cubic curve whose coefficient of x^3 is 1. The curve crosses the x-axis at A $(a,0)$ and touches the x-axis at B $(b,0)$ where a and b are positive constants such that $a < b$. The point C is a local maximum of the curve. Find the coordinate of D in terms of a and b.



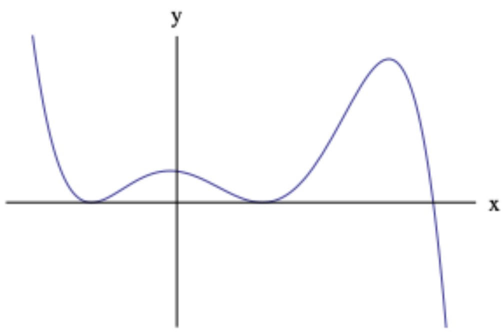
$y = (x - a)(x - b)^2$
$= (x - a)(x^2 - 2bx + b^2)$
$= x^3 - ax^2 - 2bx^2 + 2abx + b^2x - ab^2$

$\frac{dy}{dx} = 3x^2 - 2ax - 4bx + 2ab + b^2$
$= (x - b)(3x - b - 2a) = 0$

At C $\quad 3x = 2a + b \quad x = \frac{2a+b}{3}$

D $\left(\frac{2a+b}{3}, 0\right)$

7. Which one of the following equations could possibly be the graph below:



I $\quad y = (3 - x)^2(3 + x)^2(1 - x)$

II $\quad y = -x^2(x - 9)(x^2 - 3)$

III $\quad y = (x - 6)(x - 2)^2(x + 2)^2$

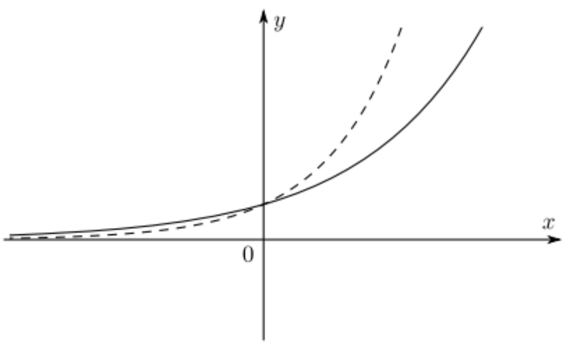
IV $\quad y = (x^2 - 1)^2(3 - x)$ $= (x-1)^2(x+1)^2(3-x)$

−ve quintic, 2 repeated roots
single root > repeated root

8. The graphs of two functions are shown.

$y = a^x$ is shown with a solid line where a is a positive real number.

$y = f(x)$ is shown with a dashed line



Which of the following could be true?

I $\quad f(x) = b^x \quad$ for some $b > a$ ✓

II $\quad f(x) = b^x \quad$ for some $b < a$ ✗

III $\quad f(x) = a^{kx} \quad$ for some $k > 1$ ✓

IV $\quad f(x) = a^{kx} \quad$ for some $k < 1$ ✗