Mock TMUA Set D: Paper 1

20 questions
75 minutes
No calculator allowed

1.

Given that $2x^2 + 11x + 5$ is a factor of $ax^3 - 17x^2 + bx - 15$

What is the value of $a + b$?

A -40 B -28 C -4 D 4 E 28 F 40

2.

The point $(-1, -1)$ lies on the curve C whose gradient function is given by

$$\frac{dy}{dx} = \frac{5x^3 - 6}{x^3} \qquad x \neq 0$$

Find an equation for C.

A $y = 5x^4 - 6x - 12$

B $y = 5x + \frac{3}{x^2} + 1$

C $y = 5x + \frac{3}{2x^4} + \frac{7}{2}$

D $y = 15x^2 + \frac{18}{x^2} - 34$

E $y = 5x - \frac{3}{x^2} + 7$

3.

A circle of radius 1cm is inscribed inside an equilateral triangle. What is the area of the triangle?

A $\frac{\sqrt{3}}{3}$ B 3 C $\pi + \frac{\sqrt{3}}{3}$ D $3\sqrt{3}$ E $6\sqrt{3}$

4.

S is the complete set of values of x which satisfy **both** the inequalities

$$x^2 - x - 6 < 0 \qquad \text{and} \qquad 4 - 3x > 1$$

The set S can also be represented as a single inequality.

Which of the following single inequalities represents the set S?

A $x^2 + x - 2 < 0$

B $x^2 + x - 2 > 0$

C $x^2 - 4x + 3 < 0$

D $x^2 - 4x + 3 > 0$

E $x^2 + 2x < 0$

F $x < 1$

5.

A cylinder has its radius reduced by 20%.
In order to keep the volume of the cylinder the same, the height needs to be increased by:

A 20% B 40% C 56.25% D 62.5% E 64%

6.

An arithmetic progression and a convergent geometric progression each have first term 1.

The sum of the second terms of the two progressions is $\frac{5}{4}$.

The sum of the third terms of the two progressions is $\frac{3}{4}$.

What is the sum to infinity of the geometric progression?

A -2 B $-\frac{3}{2}$ C $-\frac{1}{2}$ D $\frac{1}{2}$ E 2

7.

How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$cos(sin x) = \frac{1}{2}$$

A 0

B 1

C 2

D 3

E infinitely many

8.

Find the area enclosed by the graph of $|2x| + |y| = 2$

A $2\sqrt{5}$ B 4 C $2\sqrt{2}$ D $\frac{5}{2}$ E 2

9.

Find the maximum value of the function $\frac{1}{9^x - 6(3^x) + 11}$

A $\frac{1}{11}$ B $\frac{1}{3}$ C $\frac{4}{11}$ D $\frac{1}{2}$ E $\frac{11}{2}$

10.

The graph of $y = x^2 - 4$ has a series of transformations applied, resulting in the graph of $y = x^2 - 2x$

Which of the following could be the series of transformations?

A a translation two units left followed by a reflection in the y-axis

B a translation one unit left followed by a reflection in the y-axis

C a translation one unit right followed by a translation three units up

D a translation one unit right followed by a translation four units up

E a reflection in the y-axis followed by a translation four units up

11.

What is the sum of the solutions to the equation $9log_2(4x^2) = \left(2log_2 4x\right)^2$ for $x > 0$

A $\frac{1}{2}$ B 1 C $\frac{1+\sqrt{2}}{2}$ D $\frac{5}{2}$ E $\frac{4+\sqrt{2}}{2}$

12.

A triangle ABC is to be drawn with the following measurements.

$AB = \sqrt{7}\ a$ $BC = 2a$ angle $BAC = \theta°$

Of the two possible triangles that could be drawn, the area of the larger triangle and the area of the smaller triangle are in the ratio 3:1

What is the value of $cos\theta$?

A $\frac{2}{\sqrt{7}}$ B $\frac{\sqrt{3}}{\sqrt{7}}$ C $\frac{4}{\sqrt{7}}$ D $\frac{3}{7}$ E $\frac{4}{7}$

13.

C_1 and C_2 are circles defined by the equations

$$(x-12)^2+y^2=36 \quad \text{and} \quad (x+13)^2+y^2=81\ .$$

Find the length of the shortest line segment PQ which is tangent to C_1 at P and tangent to C_2 at Q.

A 15 B 16 C 18 D 20 E $\sqrt{616}$

14.

In the simplified expansion of $(2+5x)^9$.

How many of the terms have a coefficient that is divisible by 60?

A 6 B 7 C 8 D 9 E 10

15.

It is given that $y=(1-2\sin x)\cos 2x$ for $0<x<2\pi$.

The fraction of the interval $0<x<2\pi$ for which $y<0$ is

A $\frac{1}{6}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{1}{2}$ E $\frac{2}{3}$

16.

How many real roots does the equation $y = 3x^4 - 16x^3 + 18x^2 - 5$ have?

A 1 B 2 C 3 D 4 E 5

17.

Given that the real numbers x and y satisfy the equations $4^x + 4^y = 10$

and $2(4^x) + 4^{2y} = 20$

What is the value of $x + y$?

A $log_4 10$ B 2 C 4 D $4 + log_4 10$ E 10

18.

It is given that $f(x) = x^2 - 4x + 2$.

The curves $y = f(kx)$ and $y = f(c - x)$ have the same minimum point, where $k > 0$ and $c > 0$.

Which of the following is an expression for k in terms of c ?

A $k = \dfrac{c+2}{2}$ B $k = \dfrac{2}{c-2}$ C $k = \dfrac{2}{c}$ D $k = \dfrac{c-4}{4}$ E $k = \dfrac{c}{4}$

19.

Given that $x^2 + y^2 = 1$, what is the greatest possible value of $2x + 3y$

A $\sqrt{13}$

B $\sqrt{10}$

C $\sqrt{7}$

D $\frac{5\sqrt{2}}{2}$

E 3

20.

The sum of the maximum and minimum values of the function

$f(x) = (2a)^{sinx}$ where $a > 0$ and x is real, is 4.

Find the sum of the possible values of a.

A $2 - \sqrt{3}$ B $\sqrt{3}$ C $1 + \frac{1}{2}\sqrt{3}$ D 2 E $2 + \sqrt{3}$

Mock TMUA Set D: Paper 2

20 questions

75 minutes

No calculator allowed

1.

A curve has equation $y = \dfrac{(2x-3)(x+2)}{\sqrt{x}}$ $\quad x > 0$

The equation of the tangent to the curve at $x = 1$ is

A $2y = 13x - 19$

B $2y = 13x - 7$

C $2y = x - 13$

D $y = 3x - 6$

E $y = 3x + 6$

2.

Find the coefficient of x^3y^2 in the expansion of $(1 + x + y^2)^5$

A 5 B 10 C 15 D 20 E 30

3.

A $(-2, 4)$ and C $(6, 0)$ are opposite vertices of the rhombus $ABCD$.

The vertex D lies on the y-axis. What are the coordinates of vertex B ?

A $(2, -6)$

B $(4, -5)$

C $(6, 10)$

D $(5, 8)$

E $(4, 6)$

4.

Consider the following statement:

If a positive integer N has the property that N^2 is divisible by 9, then N is divisible by 6.

Which of the following is a counterexample to this statement?

I $N = 15$ II $N = 18$ III $N = 21$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

5.

Use the trapezium rule approximation with four strips to estimate:

$$\int_0^1 16^x \, dx$$

A $\frac{31}{8}$ B $\frac{45}{8}$ C $\frac{15}{2}$ D $\frac{31}{4}$ E $\frac{45}{4}$

6.

Consider the following statements for real values of x.

A: $\int_0^{2\pi} \sin kx \, dx = 0$

B: k is an even integer

Which one of the following is correct?

A A is **necessary** but **not sufficient** for B

B A is **sufficient** but **not necessary** for B

C A is **necessary** and **sufficient** for B

D A is **not necessary** and **not sufficient** for B

7.

A student attempts to prove that there are infinitely many prime numbers as follows:

Assume that there are finitely many prime numbers, and let the largest be p (I)

Let $n = 2p + 1$ so that $n > p$ (II)

Consider $n = 2p + 1$. This is not divisible by p (III)

Therefore n is also a prime number (IV)

This contradicts the initial assumption so we conclude that there are infinitely many prime numbers (V)

Which of the following best describes this proof?

A The statement is not true and there is an error in the proof in line (I)

B The statement is not true and there is an error in the proof in line (II)

C The statement is true but there is an error in the proof in line (III)

D The statement is true but there is an error in the proof in line (IV)

E The statement is true but there is an error in the proof in line (V)

F The statement is true and the proof is completely correct.

8.

Simplify the following expression $log_a 2 \times log_a 4 \times log_a 8 \times \ldots \times log_a 2^n$

A $log_a\left(2(2^n - 1)\right)$

B $log_a\left(2^{n-1} + 1\right)$

C $n!(log_a 2)^n$

D $n!(log_a 2^n)$

E $n(n!)(log_a 2)$

9.

A set of cards has a single letter printed on the front and a single number printed on the back.

Five of these cards are laid on the table so that only one side of each card is visible.

The cards show the following: 5 E 7 F 9

Lara states that all cards with a vowel on the front have a prime number on the back.

Which cards do you need to turn over in order to check if this statement is true?

A all of the cards

B cards E, 5 and 7

C card E only

D cards F and 9

E cards E and 9

10.

Consider the set of integers of the form $3^n - 2n - 1$, where n is a positive integer greater than 1.

Which of the following statements are necessarily true?

I Integers in this set are even only if n is odd.

II Integers in this set are always 2 more or 2 less than a multiple of 6.

III Integers in this set are always a multiple of 4.

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

11.

Given that $f\big(x - f(x)\big) = x$ and $f(a) = b$

which of the following is true?

A $f(b) = a - b$

B $f(-a) = a - b$

C $f(-b) = -a$

D $f(-a) = -b$

E $f(-b) = b - a$

12.

Let P be the set of prime numbers greater than 3.

Consider the following assertion:

All members of P can be represented in the form $6n \pm 1$ (*)

Which of the following statements, taken individually is equivalent to (*)?

I A number is **not** a member of P **only if** it can **not** be written in the form $6n \pm 1$

II A **sufficient** condition for a number to be in P is that it can be written in the form $6n \pm 1$

III **If** a number can **not** be written in the form $6n \pm 1$ then it is **not** a member of P

	Statement I is equivalent to (*)	Statement II is equivalent to (*)	Statement III is equivalent to (*)
A	Yes	Yes	Yes
B	Yes	Yes	No
C	Yes	No	Yes
D	Yes	No	No
E	No	Yes	Yes
F	No	Yes	No
G	No	No	Yes
H	No	No	No

13.

Arrange the following integrals in order from smallest to largest:

P: $\int_0^4 x\left|(x-2)\right| \; dx$ Q: $\int_{-4}^0 x\left|(x+2)\right| \; dx$ R: $\int_{-4}^4 x\left|x\right| \; dx$

A PQR

B PRQ

C QPR

D QRP

E RPQ

F RQP

14.

Suppose that n is an integer such that n^3 is divisible by 5.

Which of the following statements are necessarily true?

I 5 is a factor of n

II n^3 is divisible by 125

III If k is a multiple of 5 which divides n^3, then $\frac{n^3}{k}$ is a multiple of 5

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

15.

Let x be a real number.

Which **one** of the following statements is a **sufficient** condition for **all** of the other four statements?

A $x = -1$ or $x = 1$

B $x \geq -1$ and $x \leq 1$

C $x \geq -1$ or $x \leq 1$

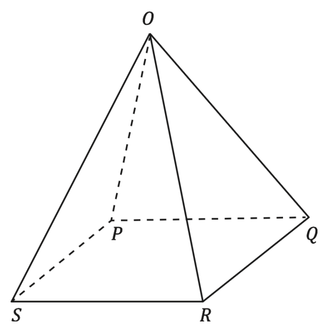
D $x \leq 1$

E $x^2 \leq 1$

16.

The diagram shows a square based pyramid with base PQRS and vertex O. All the edges of the pyramid have length 10 metres.

Find the shortest distance in metres along the outer surface of the pyramid from the midpoint of OP to the midpoint of OR.



A $5\sqrt{2}$ B $5\sqrt{3}$ C $3\sqrt{10}$ D 10 E $6\sqrt{3}$

17.

An infinite sequence of integers is such that $u_1 = -17$

and $u_{n-1} - 7 < u_n < u_{n-1} - 2$ for $n > 1$

Given that $u_k = -53$ how many different values can k take.

A 1

B 6

C 7

D 12

E 15

18.

Which of the following statements, taken independently, is/are true?

I **For all** real x, **there exists** a real y such that $y^2 = x$

II **There exists** a real y such that **for all** positive real x, $y^2 = x$

III **For all** real y, **there exists** a non-negative real x such that $y^2 = x$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

19.

Consider the quadratic graphs given by $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx + r$

If $f'(x) > g'(x)$ for all real x, which of the following is / are **necessarily** true?

I $a > p$ and $b > q$

II $a = p$ and $b > q$

III $b > q$ and $c > r$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

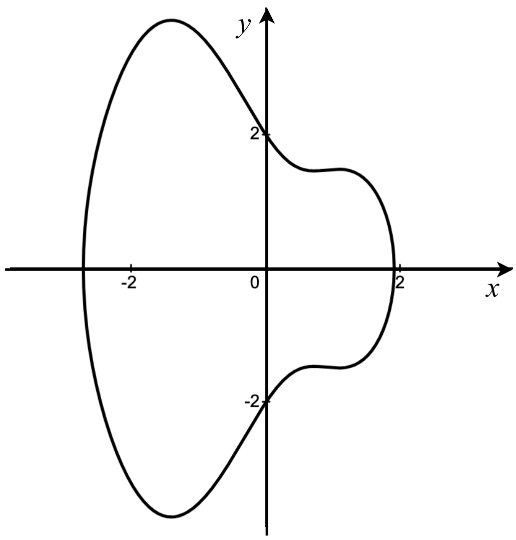
G I and III only

H I, II and III

20.

The graph shows the curve defined by

$$y^2 = 10 - 6\sin x - 6\cos^2 x - x^2$$



The point P lies on this curve and the distance of P from the origin O is defined as d.

What is the minimum value of d ?

A $\frac{2\sqrt{5}}{3}$ B $\frac{1}{2}\sqrt{10}$ C $\frac{5}{2}$ D $\sqrt{3}$ E 2